DC Circuits 1

Physics 2415 Lecture 12

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Today’s Topics

• Mention of AC
• Semiconductors and superconductors
• Battery emf, internal resistance
• Series and parallel resistances
• Kirchhoff’s rules
AC and DC

- **Batteries** provide direct current, DC: it always flows in the same direction.

- Almost all electric generators produce a voltage of **sine wave** form:

  \[ V = V_0 \sin 2\pi ft = V_0 \sin \omega t \]

- This drives an **alternating current**, AC,

  \[ I = \frac{V_0 \sin \omega t}{R} = I_0 \sin \omega t \]

and power

\[ P = VI = I^2 R = I_0^2 R \sin^2 \omega t = \left(\frac{V_0^2}{R}\right) \sin^2 \omega t \]
AC Average Power and rms Values

• The AC power \( P = \left( \frac{V_0^2}{R} \right) \sin^2 \omega t \) varies rapidly \( (\omega = 2 \pi f, f = 60 \text{ Hz here}) \), what is significant for most uses is the average power.

• The average value of \( \sin^2 \omega t \) is \( \frac{1}{2} \).

• Define \( V_{\text{rms}} \) by \( V_{\text{rms}} = \sqrt{V^2} = \frac{V_0}{\sqrt{2}} \).

• Then the average power \( \overline{P} = \frac{V_{\text{rms}}^2}{R} \).

The standard 120V AC power is \( V_{\text{rms}} = 120V \).

So the maximum voltage \( V_0 \) on a 120V line is \( 120 \times \sqrt{2} = 170V! \)
Sometimes DC *is* used for a Single Long Line

- This 3 gigawatt DC line (enough for 2 to 3 million households) transmits hydropower from the Columbia river to Los Angeles.
- At these distances, it gets tricky synchronizing the phase of AC power.
Semiconductors

- In the Bohr model of the hydrogen atom, an electron circles around a proton.
- An **n-type semiconductor** is a dielectric insulator which has been **doped**—atoms having one more electron than the insulator atoms are scattered into it.
- The extra electron circles the dopant atom, but is loosely bound because the dielectric shields the electric field—it looks like a big Bohr atom. As the temperature is raised, these electrons break away from their atoms, and become available to conduct electricity.
- **Bottom Line**: Conductivity *increases* with temperature.
Superconductors

• A superconductor has exactly zero resistivity.
• In 1911, mercury was discovered to superconduct \( R = 0 \) when cooled below 4K.
• Superconducting magnets are widely used, in MRI machines, etc.
• There are now materials superconducting above the boiling point of liquid nitrogen, making long distance transmission lines feasible.
• Superconductivity is a quantum phenomenon.
Battery emf $E$

- At the terminals inside a battery, a precise voltage is generated by the particular chemical energy exchanges taking place (electron capture or donation by molecules at the terminals).

- This voltage is called the electromotive force (even though it’s a potential energy, it *does* drive the current around a circuit), and is denoted by emf or $E$. 
The emf \( \mathcal{E} \) and *Internal* Resistance

- This chemically generated voltage \( \mathcal{E} \) also has to push the current through the battery itself.

- The battery has an *internal* resistance, usually denoted by \( r \), so for a current \( I \) in the circuit, the battery supplies to the outside world a terminal voltage

\[
V = \mathcal{E} - Ir
\]

- (This is usually a small effect and can be neglected.)
Resistances in Series

• A battery voltage $V$ pumps a steady current $I$ through 3 resistances in series, as shown.
• Think of the battery as a pump, raising the potential of charge, which then drops in the $R$'s, like a series of waterfalls $a \rightarrow b \rightarrow c \rightarrow d$.
• From Ohm’s Law, the potential drops are:
  
  $$V_{ab} = IR_1, \quad V_{bc} = IR_2, \quad V_{cd} = IR_3.$$  

• So the total drop $V = V_{ad} = V_{ab} + V_{bc} + V_{cd} = IR_1 + IR_2 + IR_3 = IR$

where the total resistance $R = R_1 + R_2 + R_3$
Resistances in Parallel

- (Convention: lines without zigzag represent wires of negligible resistance.)
- This means all three of the resistances shown have the same voltage $V$ between their ends.
- So $V = I_1 R_1 = I_2 R_2 = I_3 R_3$
- The total resistance is defined by $V = IR$.
- Now $I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = \frac{V}{R}$,

\[
\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}
\]
Clicker Question

• Which has the greater resistance,
  A. A 120V 60W bulb?
  B. A 120V 30W bulb?
Clicker Question

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  A. A 120V 60W bulb?
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Remember power \( P = VI = \frac{V^2}{R} \). \( V \) is the same for both, so lower \( R \) means higher power.
Clicker Question

• If a 60W bulb and a 100W bulb are connected in series to a 120V supply, which will be brighter?

A. The 60W bulb
B. The 100W bulb
C. They’ll be equally bright
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A. The 60W bulb
B. The 100W bulb
C. They’ll be equally bright
D. The 60W bulb has greater \( R \), so more voltage drop—and power = \( VI \), they have the same \( I \).
Remember...

- Resistances **in series** all carry the **same current**
- Resistances **in parallel** all have the **same voltage drop**
- Put this together with Ohm’s law for each resistance.
General Circuits: Kirchhoff’s Rules

• **Junction Rule:** when several wires meet at a point, the total current flowing into the point must equal the total current flowing out. Charge cannot disappear, or pile up at a point.

• **Loop Rule:** the total potential (voltage) change on following wires around a loop to your starting point must be zero.

• *(The loop rule is equivalent to saying that if you follow some random path on a hillside, and get back eventually to your starting point, your net change in height above sea level is zero.)*
All lines have resistance 1 except $dc$, which has resistance $r$.

If a voltage $v$ is applied from $a$ to $b$, which way does current flow in $dc$?

A. From $d$ to $c$
B. From $c$ to $d$
C. There is no current
All lines have resistance 1 except $dc$, which has resistance $r$. If a voltage $v$ is applied from $a$ to $b$, which way does current flow in $dc$?

- A. From $d$ to $c$
- B. From $c$ to $d$
- C. There is no current

There is no current because the situation is completely symmetrical: symmetry can sometimes simplify circuit analysis.
All lines have resistance 1 except $dc$, which has resistance $r$.

If now a voltage 10V is applied from $a$ to $c$, what is the total current flow?