Today’s Topics

- First we’ll mention capacitors
- Power usage: kWh, etc.
- The microscopic picture
- Temperature dependence of resistivity
- Drift speed and electron speed
- AC and DC
- Semiconductors and superconductors
Know This...

• Capacitors in parallel (any number) are all at the same voltage $V$.

• Capacitors in series (any number) all carry the same charge $Q$.

• Putting these facts together with $V = Q/C$ can solve a lot of problems!
Resistance and Resistivity

• To summarize: for a given material (say, copper) the resistance of a piece of uniform wire is proportional to its length $\ell$ and inversely proportional to its cross-sectional area $A$.

• This is written:

\[ R = \rho \frac{\ell}{A} \]

where $\rho$ is the resistivity.

• For copper, $\rho = 1.68 \times 10^{-8} \ \Omega \cdot m$. 
Electric Power

• Remember voltage is a measure of potential energy of electric charge, and if one coulomb drops through a potential difference of one volt it loses one joule of potential energy.

• So a current of $I$ amps flowing through a wire with $V$ volts potential difference between the ends is losing $IV$ joules per sec.

• This energy appears as heat in the wire: the electric field accelerates the electrons, which then bump into impurities and defects in the wire, and are slowed down to begin accelerating again, like a sloping pinball machine.
Power and Energy Usage

• Using Ohm’s law, we can write the power use of a resistive heater (or equivalent device, such as a bulb) in different ways:

\[ P = IV = I^2R = V^2 / R \]

• The unit is watts, meaning joules per second.

• Electric meters measure total energy usage: adding up how much power is drawn for how long, the standard unit is the kilowatt hour:

• 1 kWh = 1,000x3,600J = 3.6MJ
Microscopic Picture of Conductivity

- The total current down the wire is $I$; if we assume it’s uniform over the cross section area $A$ (which it is) there is a current density $j = I/A$. (units: amps/m$^2$)
- A constant $E$ field gives a steady current. This means the electrons are bouncing off things, like a sloping pinball machine, otherwise the current would keep accelerating.
What are the Electrons Bouncing off?

• Not the atoms! It’s found experimentally that electrons pass dozens or often hundreds of atoms before being deflected.

• Furthermore, an extremely pure crystal of copper has a very low resistance if it’s really cooled down....and the atoms are all still there.
What are the Electrons Bouncing off?

- Not the atoms!
- An extremely pure crystal of copper has a very low resistance if it’s really cooled down....
- **This is the clue**: they are deflected by thermal vibrations of the lattice—resistance increases with temperature.
- The electrons also bounce off impurities, but can pass through a pure cold lattice like light through glass... electrons are really waves!
Temperature Dependence of Resistivity

- Resistivity of metals increases approximately linearly with temperature over a wide range.
- The formula is:
  \[ \rho_T = \rho_0 \left[1 + \alpha (T - T_0)\right] \]

- \( \rho_0 \) being the resistance at some fixed \( T_0 \), and \( \alpha \) the temperature coefficient of resistivity.
- An old incandescent (not LED) bulb has a tungsten wire at about 3300K, and \( \alpha = 0.0045 \), from which \( \rho_T \approx 15 \rho_0 \) not far off proportional to temperature.
Clicker Question

• What is the resistance of a 12V, 36 Watt headlight bulb?
  A. 3 ohms
  B. 4 ohms
  C. 0.3 ohms
Clicker Answer

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- Power of 36W = $IV$, $V = 12$ so $I = 3$. Then $I = \frac{V}{R}$. 

Clicker Question

• Assume the 12V, 36 Watt headlight bulb has a tungsten filament. What is its approximate power output in the first instant it is connected, cold, to the 12V battery? \( \rho_T \approx 15 \rho_0 \).

A. 36W  
B. 2.4W  
C. 540W
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Power \(P = IV = V^2/R\): \(R\) when initially cold is \(1/15\) of \(R\) at operating temperature of 3300K.
Drift Speed

• Take a piece of copper wire, say 1mmx1mm cross section, 1m long carrying 5 amps.
• This is 1cc of Cu, about 10 gms, about $10^{23}$ conduction electrons (one per atom), about 15,000C of electron charge.
• Therefore, at 5 amps (C/sec) it takes 3000secs for an electron to drift 1m.
• **Bottom line:** the drift velocity is of order $10^{-4}$ m/sec. (it’s linear in current, and depends on wire thickness for given current, obviously.)
Drift Speed and Electron Speed

• Take a piece of copper wire, say 1mmx1mm cross section, 1m long carrying 5 amps: this wire has resistance \( R = \frac{\rho l}{A} \approx 0.02\Omega \) so from Ohm’s law \( E \approx 0.1 \text{ V/m} \).

• This field will accelerate the electrons, \( ma = eE \), approximate accn = \( 2 \times 10^{10} \) m/s\(^2\). This reaches drift velocity in about \( 0.5 \times 10^{-14} \) seconds, that must be time between collisions.

• Electron speed (from quantum mechanics) is about \( 2 \times 10^6 \) m/s, so goes of order \( 10^{-8} \) m between collisions—past dozens of atoms.
AC and DC

• **Batteries** provide direct current, DC: it always flows in the same direction.

• Almost all electric generators produce a voltage of **sine wave** form:

\[ V = V_0 \sin 2\pi ft = V_0 \sin \omega t \]

• This drives an **alternating current**, AC,

\[ I = \frac{V_0 \sin \omega t}{R} = I_0 \sin \omega t \]

and power

\[ P = VI = I^2 R = I_0^2 R \sin^2 \omega t = \left(\frac{V_0^2}{R}\right) \sin^2 \omega t \]
AC Average Power and rms Values

• The AC power \( P = \left( \frac{V_0^2}{R} \right) \sin^2 \omega t \) varies rapidly (\( \omega = 2 \pi f, f = 60 \text{ Hz} \) here), what is significant for most uses is the average power.

• The average value of \( \sin^2 \omega t \) is \( \frac{1}{2} \).

• Define \( V_{\text{rms}} \) by \( V_{\text{rms}} = \sqrt{V^2} = V_0 / \sqrt{2} \).

• Then the average power \( \bar{P} = \frac{V_{\text{rms}}^2}{R} \).

The standard 120V AC power is \( V_{\text{rms}} = 120\text{V}. \)

So the maximum voltage \( V_0 \) on a 120V line is \( 120 \times \sqrt{2} = 170\text{V}! \)
Why Bother with AC?

• Because, as we’ll discuss a little later, it’s very easy to transform from high voltage to low voltage using transformers.

• This means very high voltages can be used for longer distance transmission, low voltages for local use.
Clicker Question

- The resistivity of aluminum is 58% higher than that of copper.
- A copper high voltage line has diameter 1 cm. If it is replaced by an aluminum line of the same resistance, the aluminum line has diameter:
  A. 1.58 cm
  B. 1.27 cm
  C. 0.8 cm
  D. 0.64 cm
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Remember \( R = \rho L / A \). The power lines have the same length, the aluminum therefore needs 58% more cross-section area \( A \), from which diameter up by factor \( \sqrt{1.58} \).
High Voltage Power Lines ...

• Are made of **aluminum**—you need 58% more than copper by volume, but less than half the weight, *and* it’s about 65% cheaper per kg.
• No contest.
• Some steel may be added for strength.