Angular Momentum

Physics 1425 Lecture 21

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A New Look for $\tau = I\alpha$

- We've seen how $\tau = I\alpha$ works for a body rotating about a fixed axis.
- <u>$\tau = I\alpha$ *is* not true in general</u> if the axis of rotation is *itself* accelerating
- BUT it IS true if the axis is through the CM, and isn't changing direction!
- This is quite tricky to prove—it's in the book
- And $\boxed{\tau_{CM} = I_{CM}\alpha_{CM}}$ is often useful, as we'll see.

Forces on Hoop Rolling Down Ramp

- Take no slipping, so

$$v = R\omega, \quad a = R\alpha$$

- Translational accn $F = ma$:

$$mg\sin\theta - F_{fr} = ma$$

- Rotational accn $\tau_{CM} = I_{CM}\alpha_{CM}$:

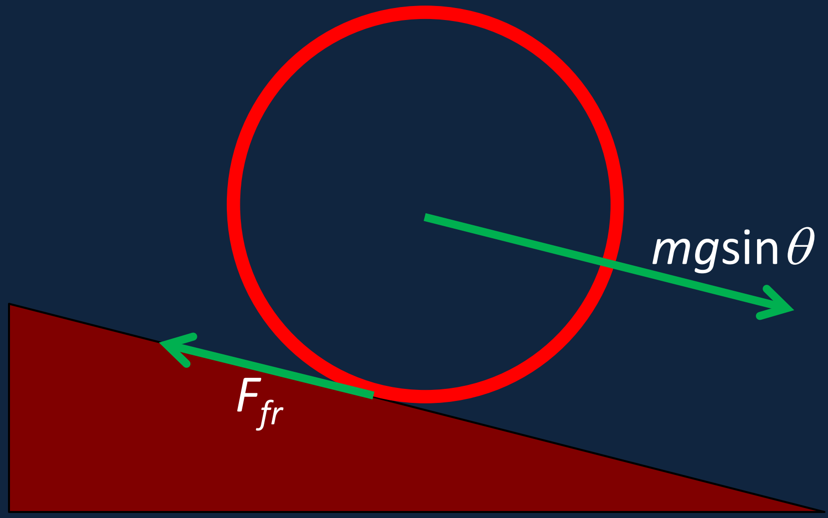
$$F_{fr}R = mR^2\alpha = mRa$$

so $F_{fr} = ma$ and

$$mg\sin\theta = 2ma,$$

- $a = (g\sin\theta)/2$:

the acceleration is one-half that of a sliding frictionless *block—and independent of mass or radius*.



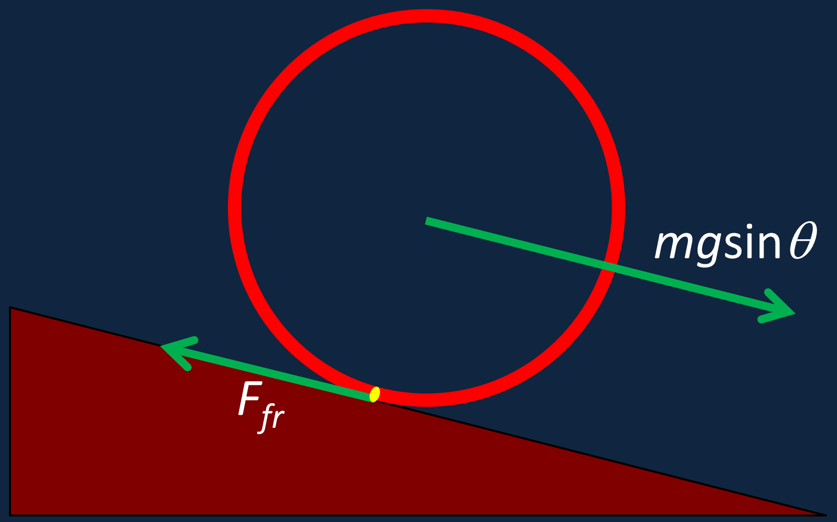
The only force having torque about the center of the hoop (its CM) is the frictional force: the total gravitational force and the normal force both act through the center.

Yet *Another* Look at That Hoop…

- Take no slipping, so

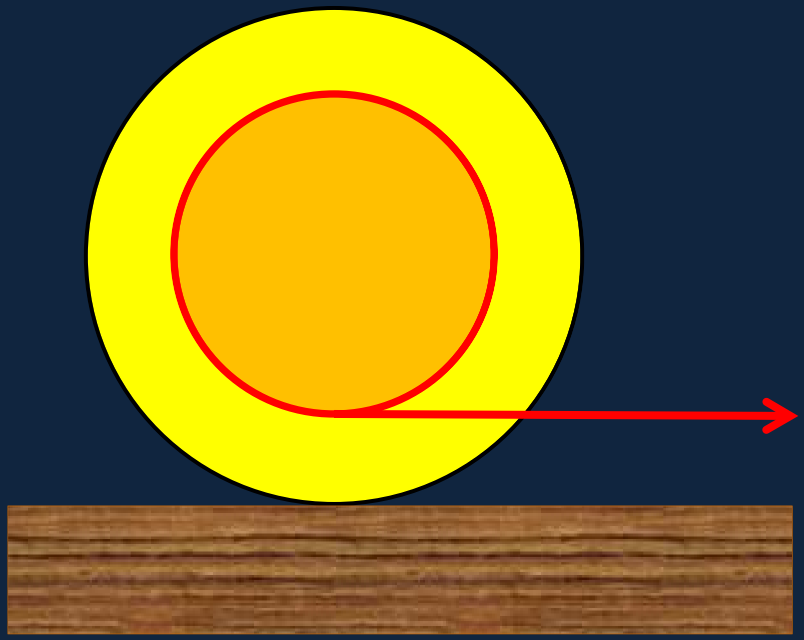
  $v = R\omega, \;\; a = R\alpha$

- Since there's no slipping, the point on the hoop in contact with the ramp is momentarily at rest, and the hoop is rotating *about that point*.
- The only torque about that point is gravity—$\tau = mgR\sin\theta$
- The moment of inertia about that point, from the parallel axis theorem, is $I_{CM} + mR^2 = 2mR^2$, so $mgR\sin\theta = 2mR^2\alpha$, and $a = \alpha/R = (g\sin\theta)/2$.



Clicker Question

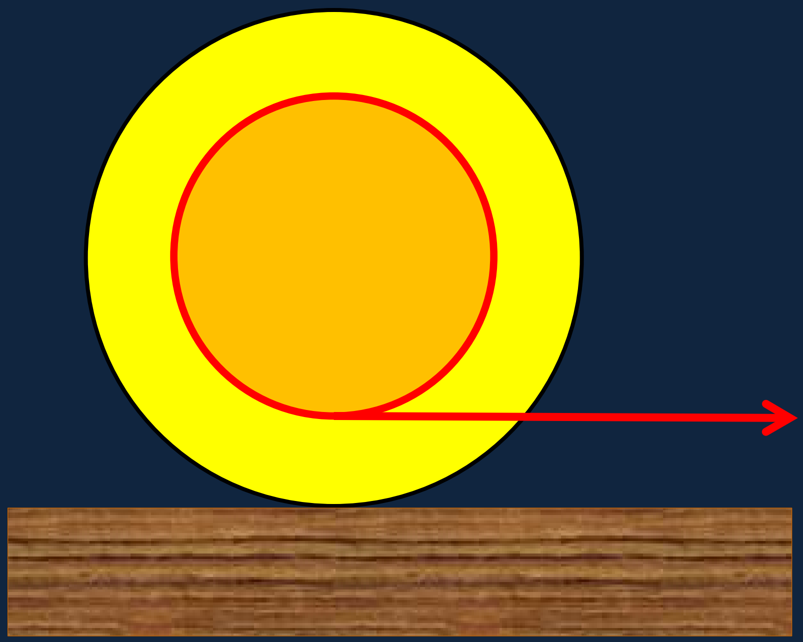
- A wooden yo-yo with red string rests on a table top. I pull the string horizontally from the bottom. What will the yo-yo do? (Assume ordinary smooth wood.)



A. Roll towards me.
B. Roll away from me.
C. Slide towards me.

Clicker Answer

- A wooden yo-yo with red string rests on a table top. I pull the string horizontally from the bottom. What will the yo-yo do? (Assume ordinary smooth wood.)

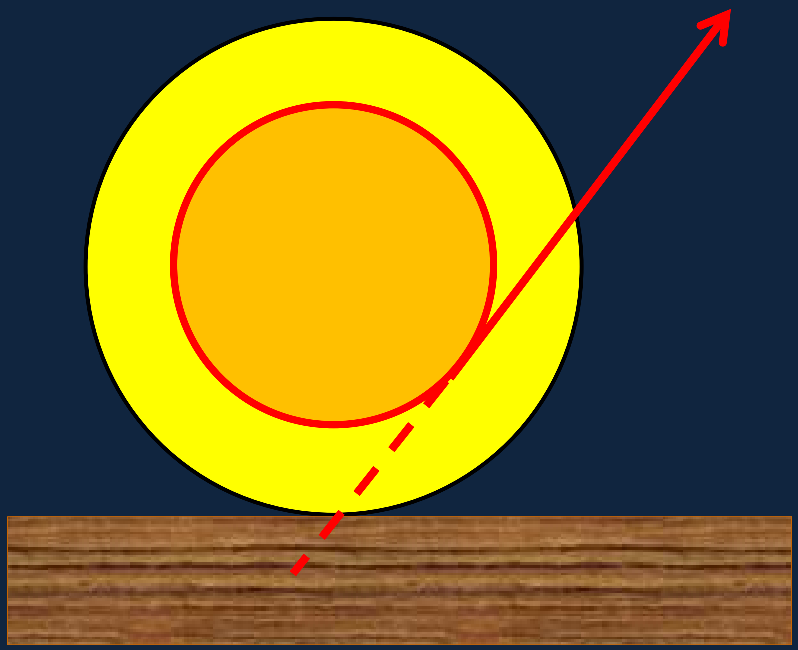


A. Roll towards me.
B. Roll away from me.
C. Slide towards me.

The key is to measure torque about the stationary point of contact of the yo-yo with the table. Clearly the torque is clockwise!

Clicker Question

- A wooden yo-yo with red string rests on a table top. I pull the string along a line that passes through the point of contact. What will the yo-yo do? (Assume ordinary smooth wood.)



A. Roll towards me.

B. Roll away from me.

C. Slide towards me.

Varying Moment of Inertia

- Recall Newton wrote his Second Law $F = dp/dt$, allowing m to vary as well as v.
- We should write the rotational version
- $\tau = d(I\omega)/dt$, and in fact varying I's are far more common than varying m's.

Clicker Question

- Assume that when she pulls herself inwards, the angular velocity increases by a factor of 3.
- What happens to 1: total angular momentum and 2: rotational kinetic energy?

A. No change, no change
B. No change, x3 increase.
C. x3 increase, x3 increase
D. x3 increase, x9 increase

Torque as a Vector

- Suppose we have a wheel spinning about a fixed axis: then $\vec{\omega}$ always points along the axis—so $d\vec{\omega}/dt$ points along the axis too.
- If we want to write a vector equation

$$\vec{\tau} = I\vec{\alpha} = Id\vec{\omega}/dt$$

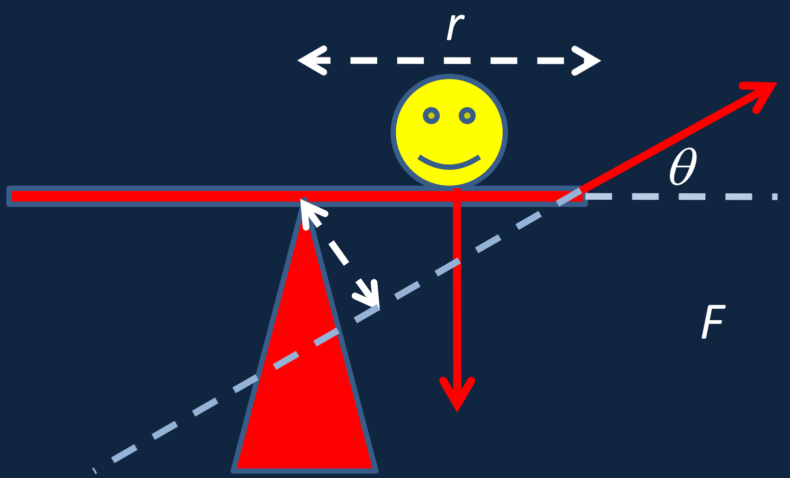
  it's clear that the vector $\vec{\tau}$ is parallel to the vector $d\vec{\omega}/dt$: so $\vec{\tau}$ points along the axis too!
- BUT this vector $\vec{\tau}$, is, remember made of two other vectors: the force $\vec{F}$ and the place $\vec{r}$ where it acts!

Recalling an Earlier Torque

- Only the component of *F* perpendicular to the arm exerts torque

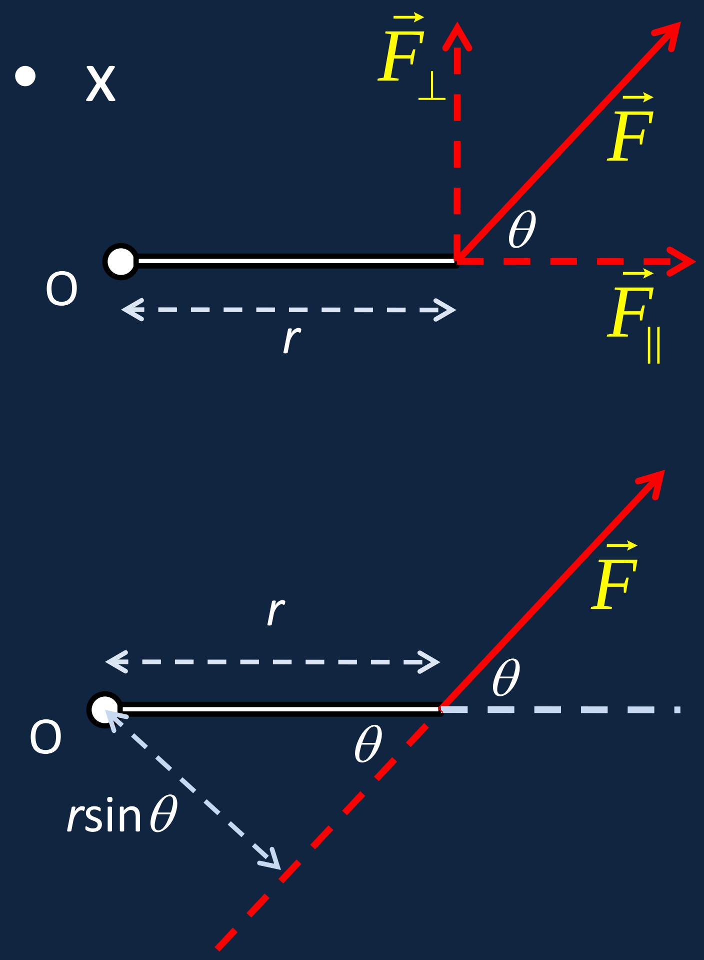
$$\tau = rF\sin\theta$$

- We can see the direction of $\vec{\tau}$ is perpendicular to both $\vec{F}, \vec{r}$ and towards us.
- We define the vector cross product $\vec{\tau} = \vec{r} \times \vec{F}$ to have this direction, and magnitude $rF\sin\theta$.



More Torque…

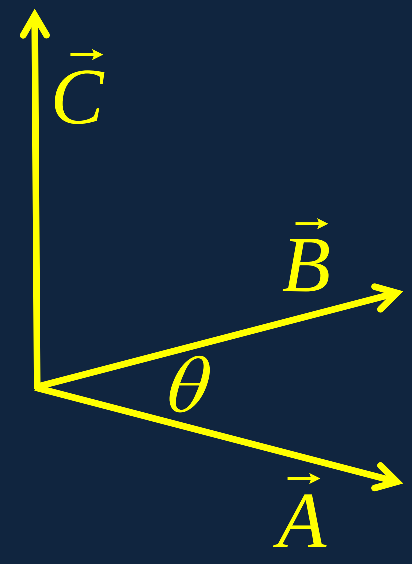
- Expressing the force vector $\vec{F}$ as a sum of components $\vec{F}_\perp$ ("fperp") perpendicular to the lever arm and $\vec{F}_\parallel$ parallel to the arm, it's clear that only $\vec{F}_\perp$ has leverage, that is, torque, about O. $\vec{F}_\perp$ has magnitude $F\sin\theta$, so $\tau = rF\sin\theta$.
- Alternatively, keep $\vec{F}$ and measure *its* lever arm about O: that's $r\sin\theta$.



Definition: The Vector Cross Product

$$\vec{C} = \vec{A} \times \vec{B}$$

- The magnitude C is $AB\sin\theta$, where θ is the angle between the vectors $\vec{A}, \vec{B}$.
- The direction of $\vec{C}$ is perpendicular to both $\vec{A}$ and $\vec{B}$, and is your right thumb direction if your curling fingers go from $\vec{A}$ to $\vec{B}$.



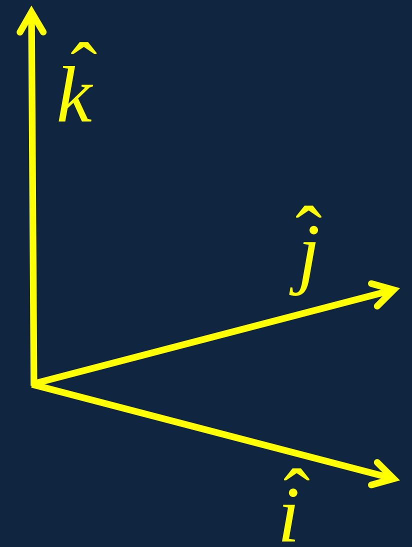
Clicker Question

Assume $\vec{A}, \vec{B}$ are nonzero vectors.
Which pair of statements below is correct?

A. The cross product depends on the order of the factors, and since both vectors are nonzero, it can never be zero.
B. Depends on order , can be zero.
C. Doesn’t depend on order, cannot be zero.
D. Doesn’tg depend on order, can be zero.

The Vector Cross Product in Components

- Recall we defined the unit vectors $\hat{i}, \hat{j}, \hat{k}$ pointing along the *x*, *y*, *z* axes respectively, and a vector can be expressed as $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$



- Now $\hat{i} \times \hat{i} = 0,\ \hat{i} \times \hat{j} = \hat{k},\ \hat{i} \times \hat{k} = -\hat{j}, \ldots$
- So

$$\vec{A} \times \vec{B} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right) \times \left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$$

$$= \hat{i}\left(A_yB_z - A_zB_y\right) + \ldots$$