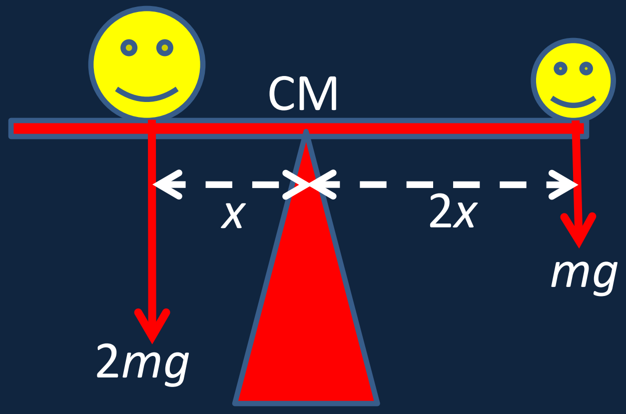
Center of Mass

Physics 1425 Lecture 17

Michael Fowler, UVa

Center of Mass and Center of Gravity

- Everyone knows that if one kid has twice the weight, the other kid must sit twice as far from the axle to balance.
- Each kid then has the same torque about the axle:
- **Torque = force x distance from the axle of the force's line of action.**
- The gravitational forces balance about the axle: it's at the center of gravity—aka the center of mass.



Center of Mass in One Dimension

- Recall the center of mass of two objects is defined by

$$\left(m_1+m_2\right)x_{\mathrm{CM}}=Mx_{\mathrm{CM}}=m_1x_1+m_2x_2$$

- Notice that if we take x_{CM} as the origin (the center of mass frame) then the equation is just

$$m_1x_1+m_2x_2=0$$

precisely the balance equation from before (one of those *x*'s is negative, of course).

CM of Several Objects in One Dimension

- The general formula is:

$$x_{\text{CM}} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i} = \frac{\sum_{i=1}^{n} m_i x_i}{M}$$

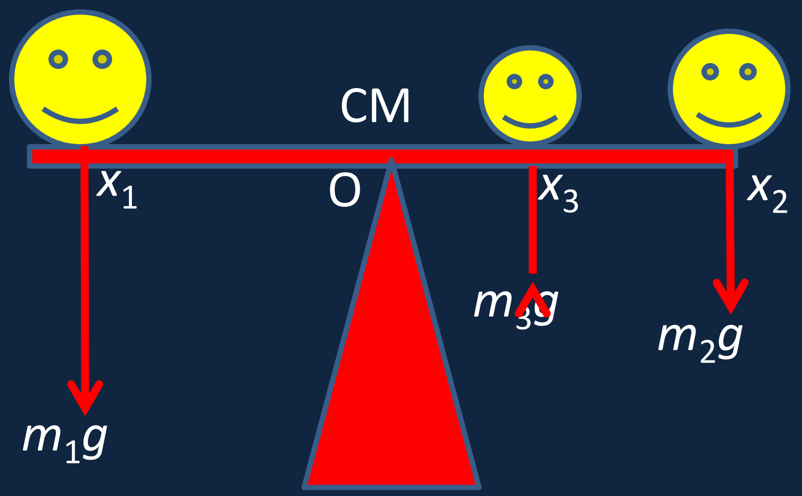
- But before putting in numbers, it's worth staring at the system to see if it's symmetric about any point!

Add Another Kid to the Seesaw…

- For the three to be in balance, the sum of the torques about the axle must be zero, so:

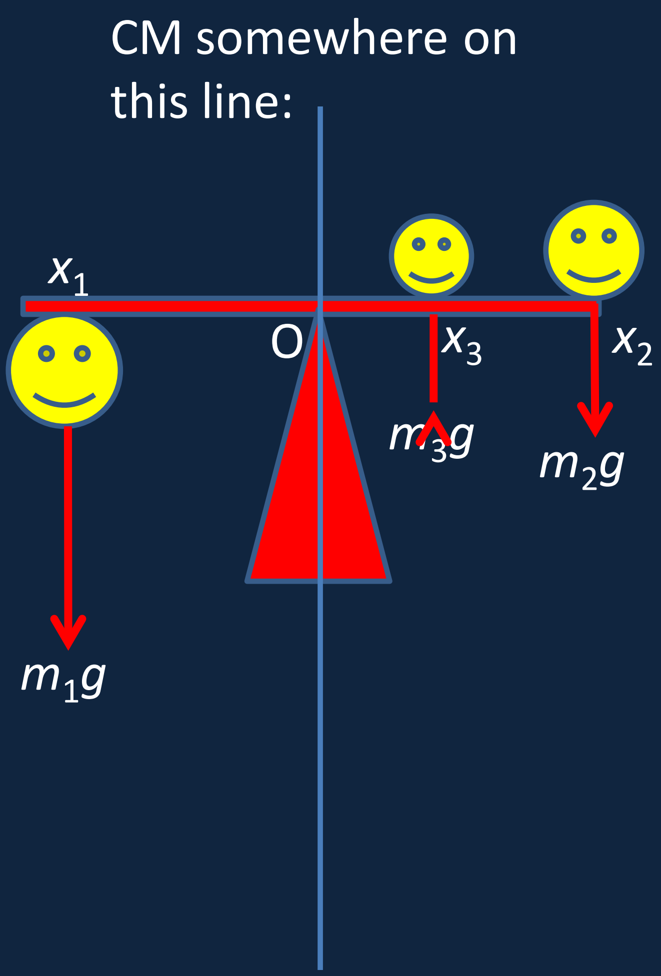
$$m_1 x_1 + m_2 x_2 + m_3 x_3 = 0$$

- That is to say, the *x* coordinate of the center of mass must be the same as the *x*-coordinate of the axle.
- This is clearly extendable to *any* number of masses ….



Some Gymnastics

- The equation
  $$m_1 x_1 + m_2 x_2 + m_3 x_3 = 0$$
  is still correct even if one kid is hanging by his hands below the seesaw!
- The center of mass is not *at* the balance point (the axle) but *is in the same vertical straight line*.



Center of Mass of a Two-Dimensional Object

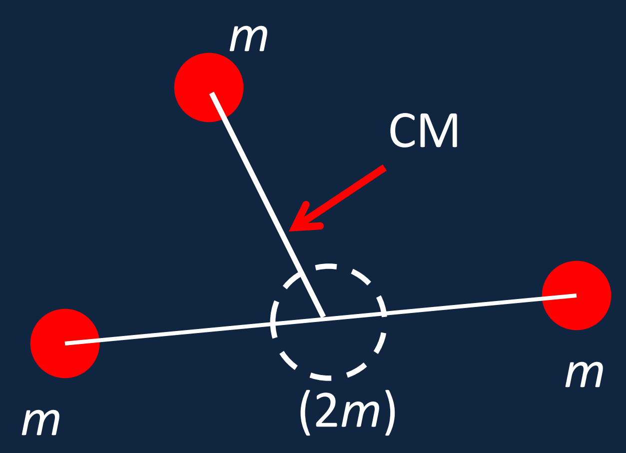
- Think of some shape cut out of cardboard.
- Hang it vertically by pushing a pin through some point.
- Think of it as made up of many small masses—when it's hanging at rest, the center of mass will be somewhere on the vertical line through the pin. Draw the line.
- Repeat with the pin somewhere else: the lines you drew meet at the CM.

Tip: if the object is symmetric about some line, the center of mass will be *on that line*!

Three Equal Masses

- If we have three equal masses at the corners of a triangle, the center of mass of two of them is the half-way point on the side joining them.
- We can replace them by a mass 2*m* at that point, then the CM of *all three masses* is on the line from the other vertex to that point, one-third of the way up.
- This is the centroid of the triangle, and is at

$$\vec{r} = \frac{\vec{r}_1 + \vec{r}_2 + \vec{r}_3}{3}$$

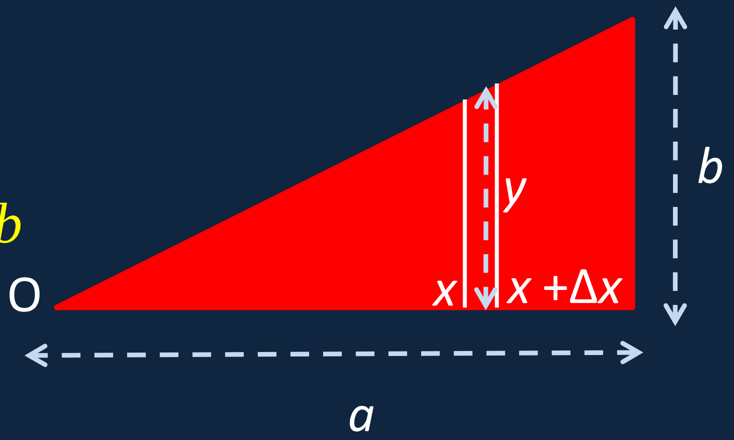


Center of Mass of a Solid Triangle

- We'll take a right-angled triangle. The x-coordinate of the CM is found by the integral generalization of the sum $$Mx_{\rm CM} = \sum_{i=1}^{n} m_i x_i$$
- If the triangle has area mass density ρ kg/m^2, the strip shown has mass $\rho y \Delta x$, and $M = ½ \rho ab$, so

$$\tfrac{1}{2}\rho ab x_{\rm CM} = \int_0^a \rho x y dx = (b/a)\int_0^a \rho x^2 dx = \tfrac{1}{3}\rho a^2 b$$

from which the CM is at (2/3)a.



- Bottom line: the CM of the solid triangle is at the same point as the CM of three equal masses at the corners!

The height y of the strip at x is given by $y/b = x/a$, from similar triangles.