Circular Motion

Physics 1425 Lecture 9

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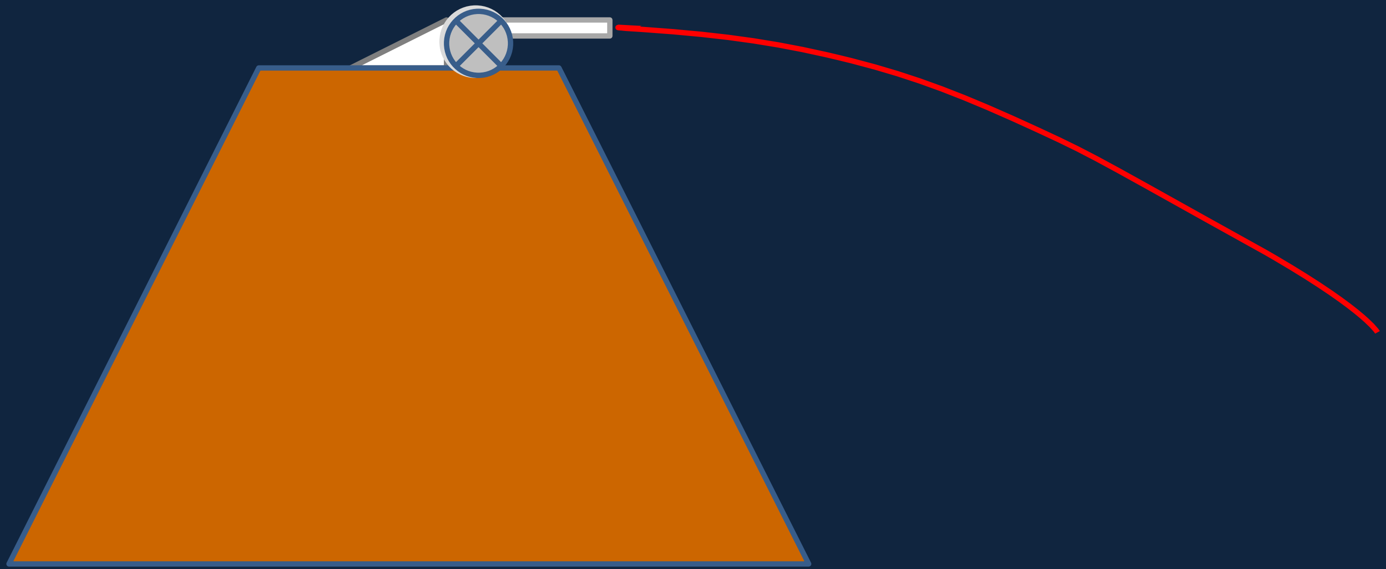
A Cannon on a Mountain

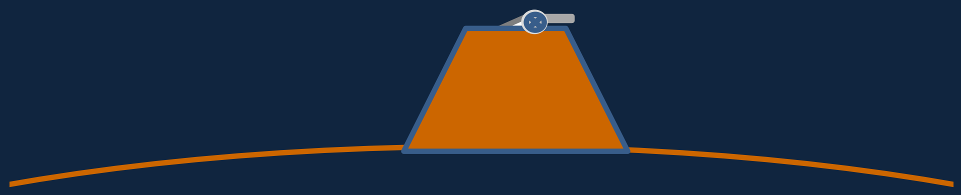
- Back to Galileo one more time… imagine a powerful cannon shooting horizontally from a high mountaintop:



- The path falls 5 m below a horizontal line in one second.

Newton's Idea: a *Really* High Mountain

- Imagine a *very* powerful cannon atop a mountain beyond the Earth's atmosphere.
- This cannonball goes so far we have to include the Earth's curvature in our calculations!
- The Earth's surface drops 5m below a horizontal plane on traveling 8 km.



- So what happens if the cannonball goes at 8 km/sec?

After Traveling 8 Kilometers in 1 second…

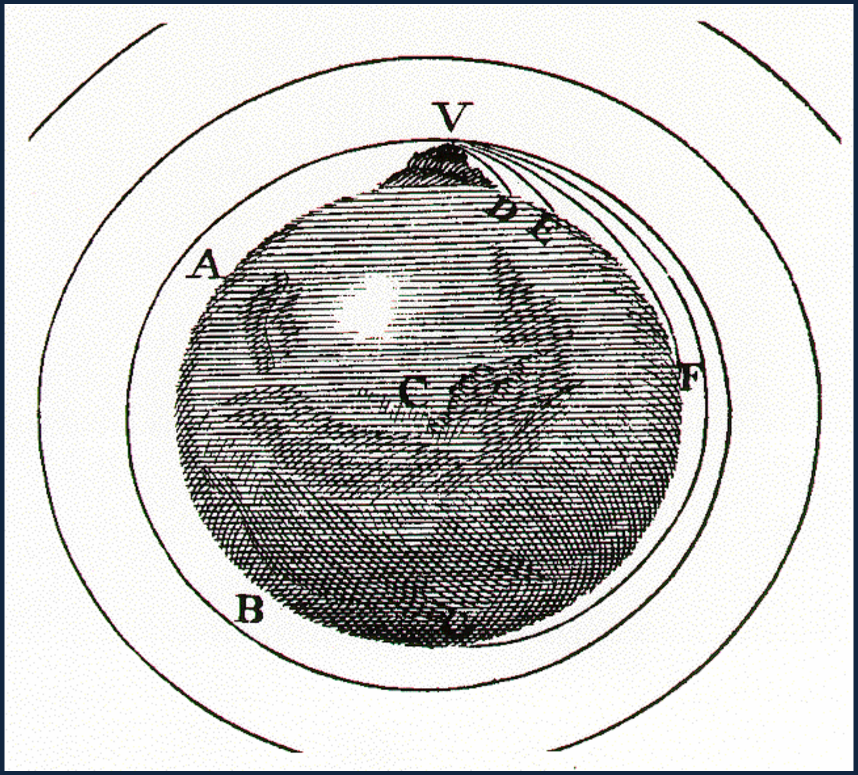
- The cannonball's velocity has slightly changed direction, adding about *g* = 10 m/sec downwards, so the angle of change is given by $\tan\theta = 10/8000$.

- BUT the Earth's surface underneath the cannonball has turned by *precisely* the same amount—and so has the direction of gravity!

- The cannonball finds itself in **exactly the same situation it began in**: moving parallel to the surface, perpendicular to gravity, at the same height.
- **So what happens next?**

Newton's Own Picture

- Newton realized that at the right initial speed, above the atmosphere, the cannonball would circle indefinitely, accelerating towards the Earth constantly, *but* staying at the same height.

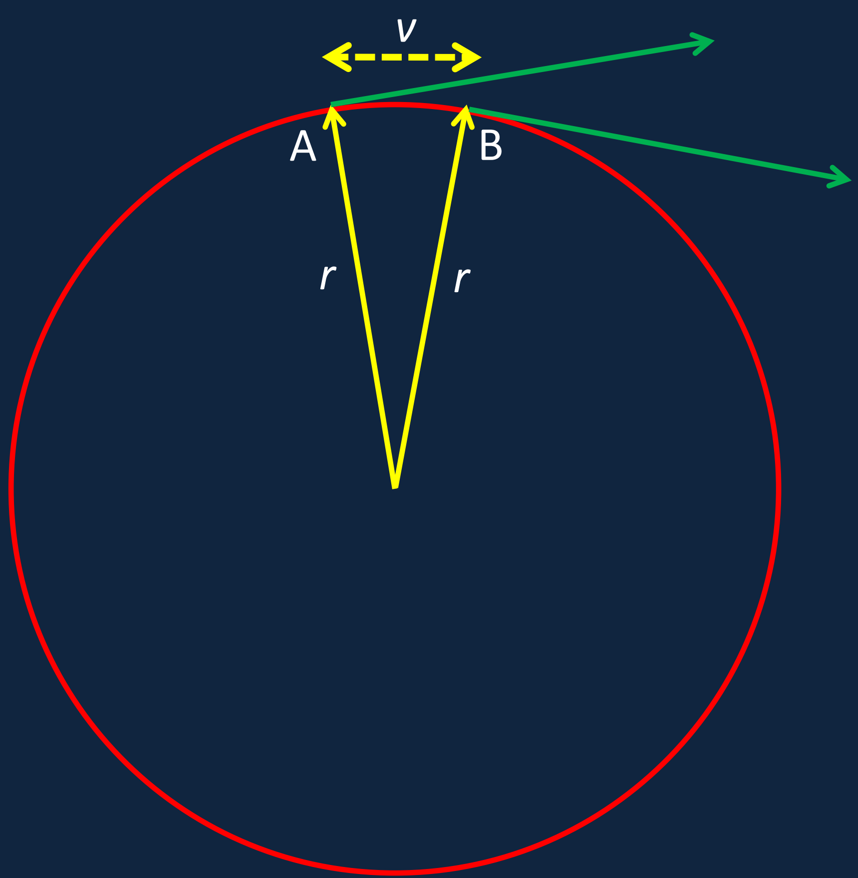


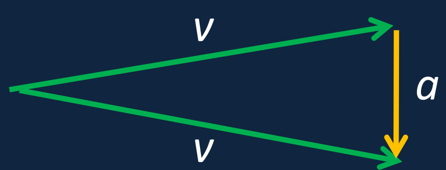
Link to animation

Acceleration in Steady Circular Motion

- A ball circling at constant speed v goes from A to B in one second: in the limit of a small angle, distance AB = v.
- The velocity vectors are perpendicular to the position vectors, so they turn through the same angle.
- Hence a/v = dist AB/r = v/r, That is,

$$a = v^2/r$$





Dynamics of Circular Motion

- Constant speed circular motion has acceleration of constant magnitude but always changing direction: it points at all times to the center of the circle.
- So from $\vec{F} = m\vec{a}$, to maintain steady circular motion, a body must experience a net force of constant magnitude directed always to the center of the circle.

Low Earth Orbit

- Newton had discovered the path of a satellite in low Earth orbit!
- For a circular orbit close to Earth's surface, $\vec{F} = m\vec{a}$ is just $mg = mv^2 / r$.
- So the speed for low orbit motion is $v^2 = rg$: that's 8 km/sec, round the Earth in 80 minutes.
- Newton's next question: why does the *Moon* circle the Earth? Could it be the same reason? The force of gravity extends to the Moon?

The Moon's Orbit

- Assuming the Moon's circular orbit *is* a result of gravitational pulling from the Earth, does the Moon feel $F = mg$ as we do?

- That's easy to check: Newton found the Moon's acceleration, using v^2/r. The distance was known (384,000,000m), the speed in orbit is close to 1 km/sec (it goes around in one month) …

- **Bottom Line: $v^2/r = 0.0026 m/s^2 = g/3600$.**

Basic Moon Facts

- The apple accelerates downwards 3,600 times faster than the Moon.
- The Moon is 60 times further from the center of the Earth than the apple is.
- What did Newton conclude from those facts?

http://dallasvintageshop.com/?p=1097

The Inverse Square Law of Gravity

- If the force of gravity has decreased by a factor of 3,600 on increasing the distance from the center of the Earth by a factor of 60, Newton concluded that the Earth's gravitational force

$$F \propto \frac{1}{r^2}$$

- This is the <u>inverse square law</u> of gravity.
- We'll get back to gravity in the next lecture …

Let's look at some different circular motion…

But why mess with toys—just do it!

Is this for real?

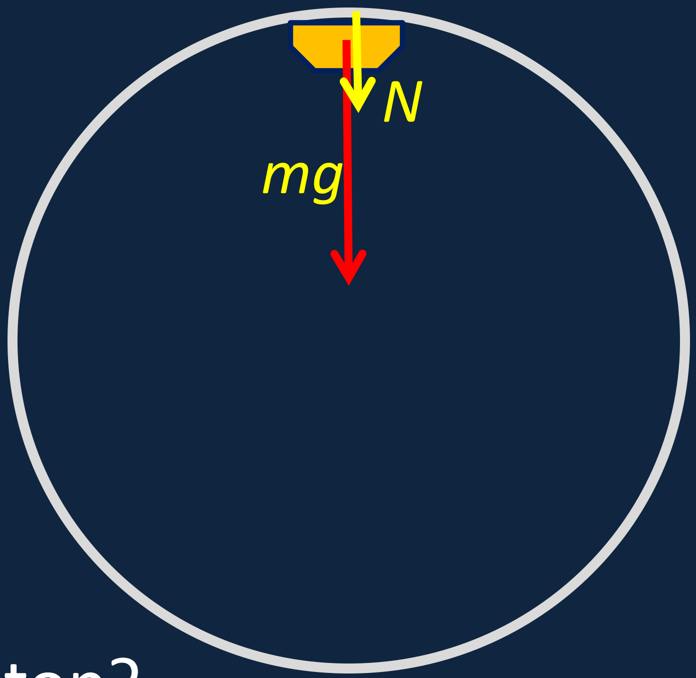
http://www.youtube.com/watch?v=wiZoVAZGgsw&NR=1

What is the Normal Force from the Track?

- At the top, $\vec{F} = m\vec{a}$ is just

$$N + mg = \frac{mv_{\text{top}}^2}{r}$$

all directed downwards.



If $v_{\text{top}} = \sqrt{rg}, \quad N = 0.$

What happens for lower *v* at the top?

http://www.youtube.com/watch?v=rVYUevr2Rag&NR=1
http://www.youtube.com/watch?v=KNu7vkfhiW0&NR=1

Clicker Question

If the loop track has a radius of 6 meters, approximately how fast must the car be going at the top to stay on the track?

A. About 8 m/s (18 mph)
B. About 12 m/s
C. About 16 m/s
D. About 24 m/s

Clicker Question Answer

If the loop track has a radius of 6 meters, approximately how fast must the car be going at the top to stay on the track?

A. About 8 m/s (18 mph) ← $v^2 = rg = 60$
B. About 12 m/s
C. About 16 m/s
D. About 24 m/s

What's the Normal Force at the *Bottom*?

- Galileo would have understood: the speed gained swinging round the track from top to bottom is the same as the speed gained if you'd just fallen directly—and that would have been with acceleration *g*, a distance 2*r*,

$$v_{\text{bottom}}^2 = v_{\text{top}}^2 + 2ax = v_{\text{top}}^2 + 4gr$$

- Recall $v_{\text{top}}^2 = gr$ to make it around, so $v_{\text{bottom}}^2 = 5rg$, if it's going just fast enough to stay on track.

Clicker Question

- If the driver has mass m, and the speed is just high enough to stay in contact with the track coasting, what is the normal force the seat exerts on him as the car enters the bottom of the loop?

A. mg

B. $2mg$

C. $5mg$

D. $6mg$

Clicker Question Answer

- If the driver has mass *m*, what is the normal force the seat exerts on him as the car enters the bottom of the loop?

A. *mg*

B. 2*mg*

C. 5*mg*

- 6*mg* ← Recall that $v_{\text{bottom}}^2 = 5rg$, so he is accelerating upwards at $v^2/r = 5g$. His weight is *mg* downwards, so the upward normal force = 6*mg*. (And notice this 6*mg* *doesn't* depend on *r*!)

Centripetal and Centrifugal…

Circular motion is maintained by a force directed to the center of the circle: this is called the centripetal force.

But if the frame of reference is itself rotating (and hence an accelerating, noninertial frame) Newton's Laws are different: in *that* frame, there *is* an apparent force tugging outwards from the center—the centrifugal force.

(Note: We'll avoid that frame!)

http://imgs.xkcd.com/comics/centrifugal_force.png