One-Dimensional Motion: Displacement, Velocity, Acceleration

Physics 1425 Lecture 2

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Today's Topics

- The previous lecture covered measurement, units, accuracy, significant figures, estimation.
- Today we'll focus on motion along a straight line: distance and displacement, average and instantaneous velocity and acceleration, the importance of sign.
- We'll discuss the important constant acceleration formulas.

Kinematics: Describing Motion

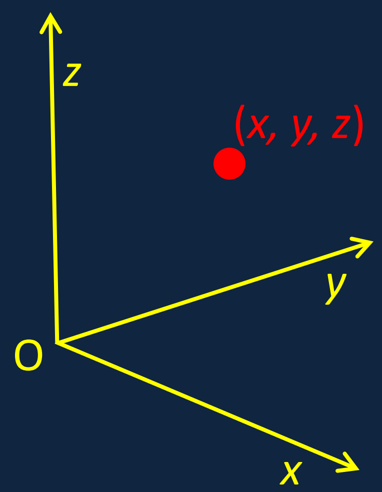
Kinematics describes *quantitatively* how a body moves through space.

We'll begin by treating the body as rigid and non-rotating, so we can fully describe the motion by following its center.

Dynamics accounts for the observed motion in terms of forces, etc. We'll get to that later.

Measuring Motion: a Frame of Reference

Frame of reference:



The frame can be envisioned as three meter sticks at right angles to each other, like the beginning of the frame of a structure.

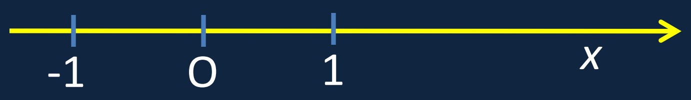
To measure motion, we must first measure position.

We measure position relative to some fixed point O, called the origin.

We give the ball's location as (x, y, z): we reach it from O by moving x meters along the x-axis, followed by y parallel to the y-axis and finally z parallel to the z-axis.

One-Dimensional Motion: Distance Traveled and Displacement

- The frame of reference in one dimension is just a line!
- Think of a straight road.



This time we've made explicit that the *x*-axis also extends in the *negative* direction, so we can label all possible positions.

- Driving a car, the distance traveled is what the odometer reads.
- The displacement is the difference $x_2 - x_1$ from where you started (x_1) to where you finished (x_2).
- They're only the same *if you only go in one direction*!

Distance and Displacement

- Take I-64 as straight, count Richmond direction as positive.
- Drive to Richmond: distance = 120 km (approx), displacement = 120 km.
- Drive to Richmond and half way back:
- Distance = 180 km, displacement = 60 km.
- Drive to closest Skyline Drive entrance:
- Distance = 35 km, displacement = -35 km.

Displacement is a Vector!

- A displacement along a straight line has magnitude and direction: + or – .   That means it's a vector.

- If the displacement $\Delta x = x_2 - x_1$, magnitude is written

$$|\Delta x| = |x_2 - x_1|.$$

- Direction is indicated by attaching an arrowhead to the displacement :

Charlottesville to Richmond

Charlottesville to Skyline Drive

Average Speed and Average Velocity

- Average speed = distance car driven/time taken.
- Average velocity = displacement/time taken

so average velocity is a vector! It can be negative.

- Formula for average velocity: $\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$

- Example: round trip to Richmond.

Average speed = 60 mph ≈ 27 m/sec.

Average *velocity* = zero!

Instantaneous Velocity

- That's the velocity at one moment of time: car speedometer gives instantaneous speed.
- To find this, need to find car's displacement in a very short time interval (to minimize speed variation).
- Mathematically, we write: $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}.$

This "lim" just means taking a succession of shorter and shorter time intervals at the moment in time.

Average Trip Speed

You drive 60 miles at 60 mph, then 60 miles at 30 mph. What was your average speed?

A. 40 mph
B. 45 mph
C. 47.5 mph

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You drive 60 miles at 60 mph, then 60 miles at 30 mph. What was your average speed?

A. 40 mph ←
B. 45 mph
C. 47.5 mph

Total distance = 120 miles, time taken = 3 hours.

Acceleration

- Average acceleration = velocity change/time taken

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

- Notice that acceleration relates to change in velocity exactly as velocity relates to change in displacement.

- Velocity is a vector, so acceleration is a vector.

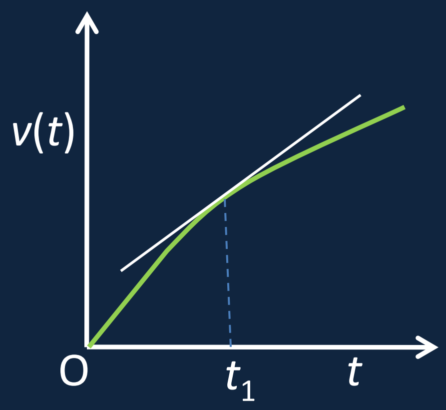
- Taking displacement towards Richmond as positive:
- *Slowing down* while driving *to Richmond*: negative acceleration.
- *Speeding up* driving *to Skyline Drive*: *also* negative acceleration!

Instantaneous Acceleration

- This is just like the definition of instantaneous velocity:
- The instantaneous acceleration

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}.$$

- The acceleration at time t_1 is the slope of the velocity graph $v(t)$ at that time.



Our Units for One-Dimensional Motion

- Displacement: meters (can be positive or negative)
- Velocity = rate of change of displacement, units: Meters per second, written m/s or $m.sec^{-1}$.
- Acceleration = rate of change of velocity, units: Meters per second per second, written m/s^2 or $m.sec^{-2}$.

Constant Acceleration

- Constant acceleration means the rate of change of velocity is constant.

$$\frac{dv}{dt} = a = \text{ constant.}$$

- The solution to this equation is

$$v = v_0 + at.$$

- Check with an example: a car traveling at 10 m/s accelerates steadily at 2 m/s^2. How fast is it going after 2 secs? After 4 secs?

Distance Moved at Constant Acceleration

- At constant acceleration,

$$\frac{dx}{dt} = v(t) = v_0 + at.$$

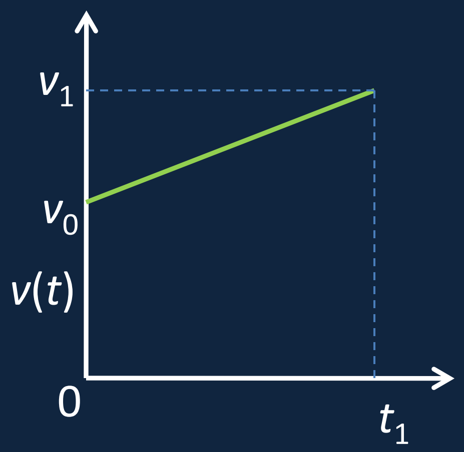
- The solution of this equation is

$$x(t) = x_0 + v_0 t + \tfrac{1}{2} a t^2.$$

- Here x_0 is the beginning position, v_0 the beginning velocity, a the constant acceleration.
- *Exercise*: check this by finding dx/dt.

More about Constant Acceleration…

- At constant acceleration, the graph of velocity as a function of time $v(t) = v_0 + at$ is a straight line:



- If $v = v_0$ at $t = 0$, and $v = v_1$ at $t = t_1$, the average velocity over the time interval 0 to t_1 is

$$\bar{v} = \frac{v_0 + v_1}{2}.$$

- IMPORTANT! This formula is unlikely to be correct at *nonconstant* acceleration.

Constant Acceleration Formulas

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

$$\bar{v} = \frac{v_0 + v_1}{2}$$

$$v^2 = v_0^2 + 2a\left(x - x_0\right)$$

These formulas are worth memorizing: the last one is simply derived by eliminating *t* between the first two.

The picture below shows time (4.56 secs) and speed (321 mph) for a standing start quarter mile at Indianapolis.
Assuming constant acceleration, what was the approximate horizontal g-force on the driver?

a. 0.3g
b. 0.8g
c. 1.5g
d. 3g
e. 5g