Projectiles

Physics 1425 Lecture 5

Reminder: Galileo's Laws of Motion

- Neglecting air resistance and friction, Galileo claimed:

1. Horizontal motion: (ball rolling on table) an object would continue to move at constant velocity unless pushed.

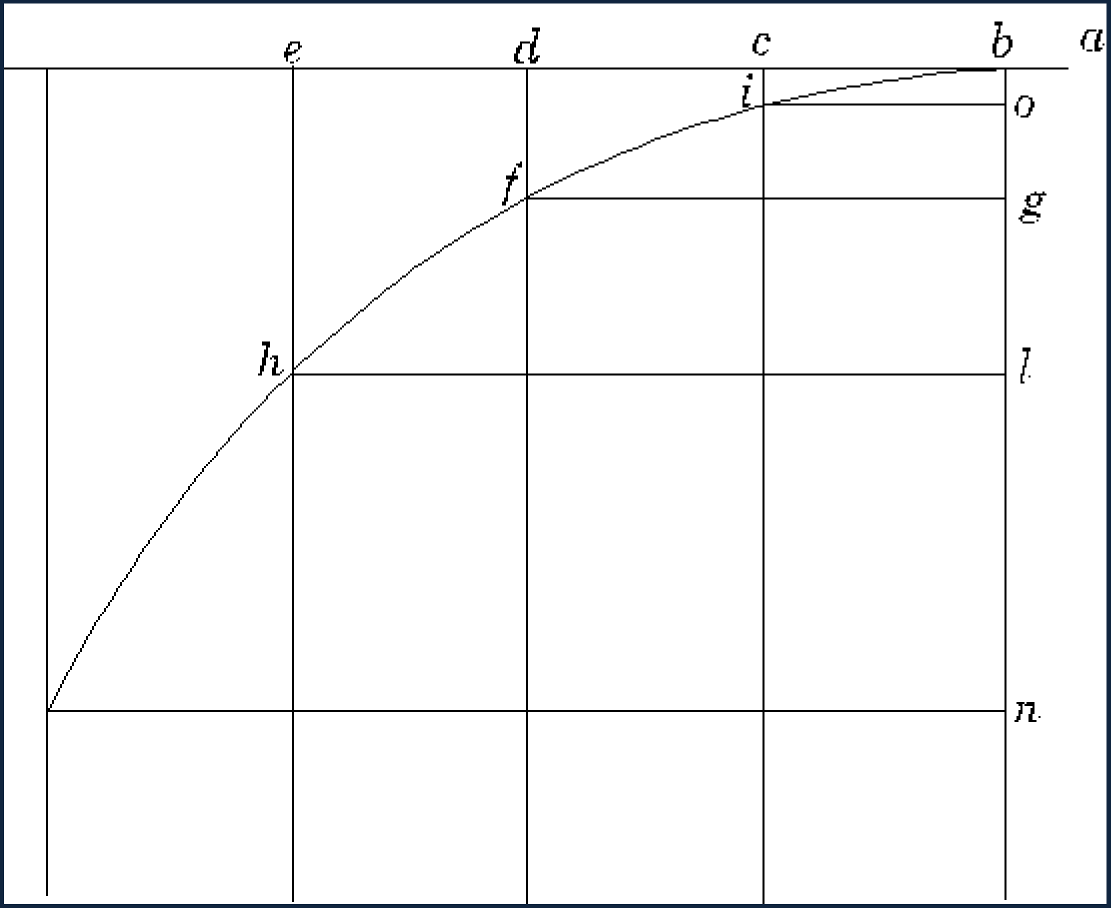
2. Vertical motion: a falling body would accelerate downwards at a uniform rate.

Galileo's Law for Projectiles

- He asked himself: what would happen if the ball rolled off the table?
- He claimed (and established experimentally) that the balls uniform horizontal motion would continue as before –
- BUT natural vertical falling motion would be added!
- He termed the result "compound motion".

Here's Galileo's own picture …

and here's a link to an animation.



Equation for the Trajectory

- Taking $t = 0$ to be when the ball rolls off the edge, and the origin O at that point,

$$x = v_{0x}t$$

$$y = -\tfrac{1}{2}gt^2$$

from which $y = -\left(g / 2v_{0x}^2\right)x^2$.

The standard parabola equation is $y = ax^2$, so this is half of an upside-down parabola.

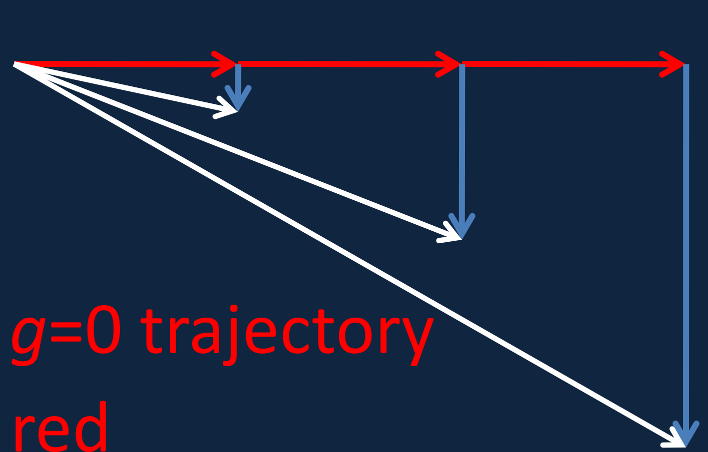
Clicker Question

- Suppose that as the ball rolling across the table goes over the edge, it touches another ball that was just balanced on the edge. Assume the first ball’s trajectory is not changed, the second ball falls vertically down.

A. The rolling ball hits the ground first
B. The dropping ball gets there first
C. They hit the ground at the same time

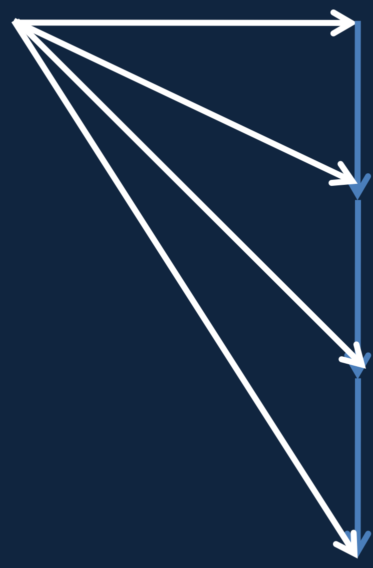
Car Goes Horizontally Over Cliff

- Position at 1 second intervals (v_0 = 20 m/s)



- *g*=0 trajectory in red
- Taking g = 10, positions are (0, 0), (20, -5), (40, -20), (60, -45).

- Velocities and Speeds at 1 second intervals:



- Speeds are approx: 20, 22, 28, 36 m/s.

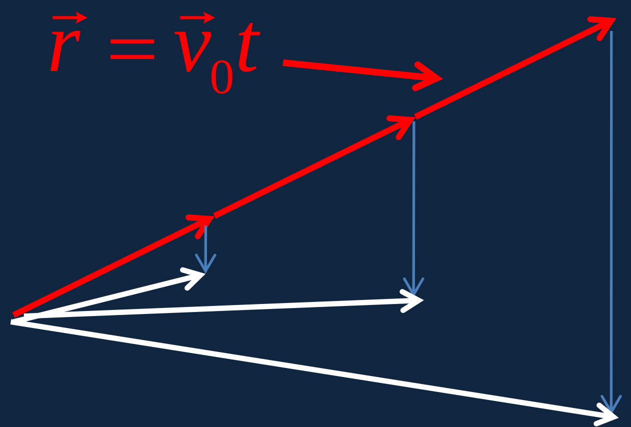
Full Projectile Path

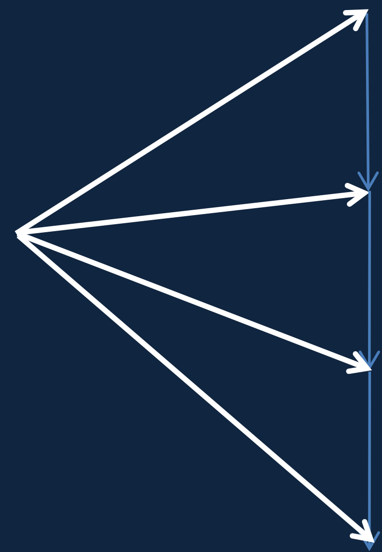
- A projectile is shot at some upward angle from the origin: see animation.
- Galileo tells us the horizontal motion is just steady velocity, the vertical motion is the same as that of a ball thrown directly upwards.
- Therefore $x = v_{0x}t,\ \ y = v_{0y}t - \frac{1}{2}gt^2.$
- Eliminating *t* gives a parabolic curve through O:

$$y = \left(v_{0y} / v_{0x}\right)x - \left(g / 2v_{0x}^2\right)x^2.$$

Vector Picture of Projectile Motion

$$\vec{r} = \vec{v}_0 t - \tfrac{1}{2}\vec{g}t^2$$





Position at 1 second intervals (notice it falls below straight line: the *g* =0 trajectory).

Velocities and Speeds at 1 second intervals.

ConcepTest 3.5 Dropping a Package

You drop a package from a plane flying at constant speed in a straight line. Without air resistance, the package will:

1) quickly lag behind the plane while falling
2) remain vertically under the plane while falling
3) move ahead of the plane while falling
4) not fall at all

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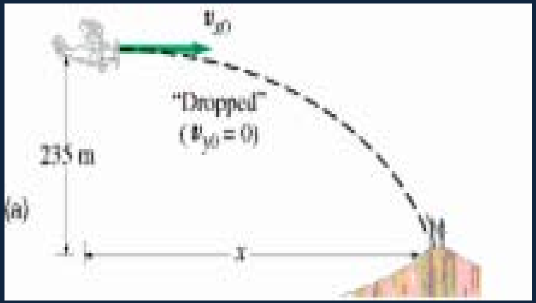
Both the plane and the package have the ***same horizontal velocity*** at the moment of release. They will ***maintain*** this velocity in the ***x-direction***, so they stay aligned.



Follow-up: what would happen if air resistance is present?

ConcepTest 3.6c Dropping the Ball III

A projectile is launched from the ground at an angle of 30°. At what point in its trajectory does this projectile have the least speed?

1) just after it is launched
2) at the highest point in its flight
3) just before it hits the ground
4) halfway between the ground and the highest point
5) speed is always constant

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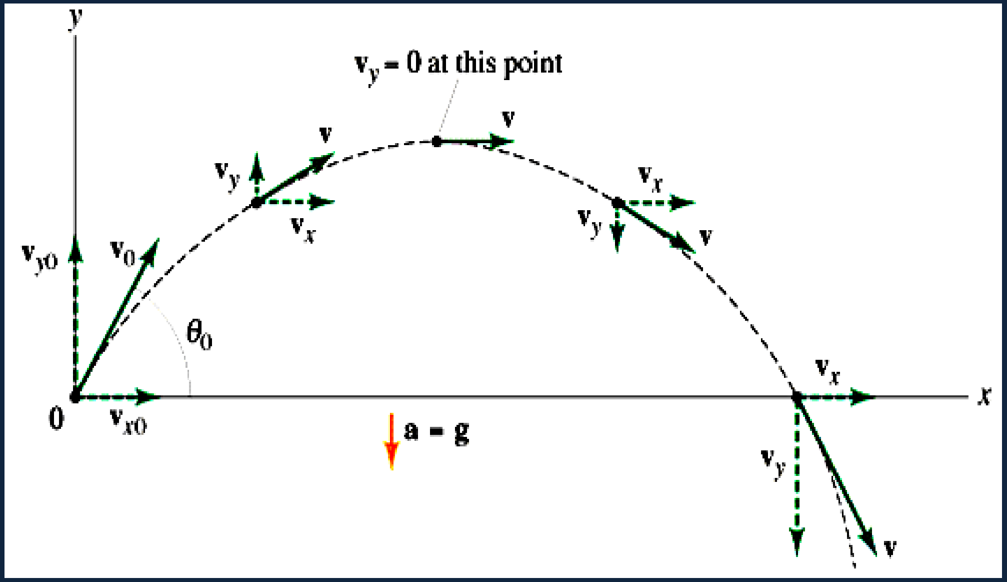
The speed is **smallest** at the **highest point** of its flight path because the *y*-component of the velocity is zero.



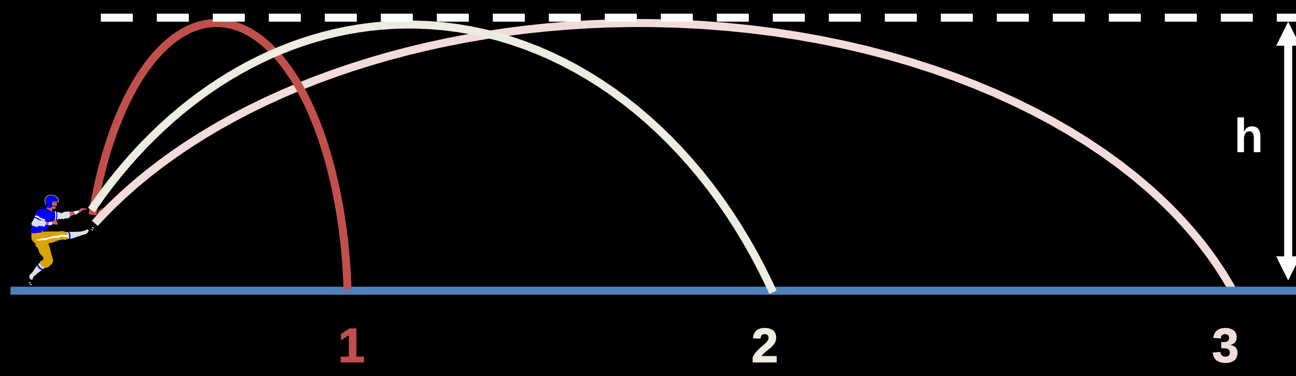
Hang Time

- How long does it take the ball to go up and come back down again?
- Initial upward velocity v_{0y}, upward velocity changes by $-g$ each second, back down at $-v_{0y}$, so $gt = 2v_{oy}$
- Hang time: $t = \dfrac{2v_{0y}}{g}$

http://curlyr.blogspot.com/2008_09_01_archive.html

ConcepTest 3.7a Punts I

Which of the three punts has the longest hang time?



4) all have the same hang time

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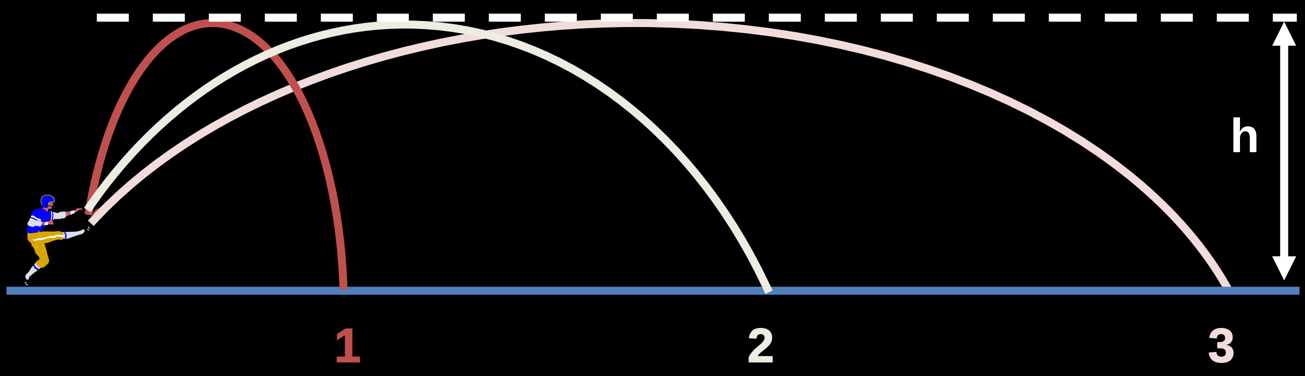


4) all have the same hang time

The time in the air is determined by the ***vertical motion***! Because all of the punts reach the **same height**, they all stay in the air for the **same time**.

Follow-up: which one had the greater initial velocity?

Range

- Given the initial velocity

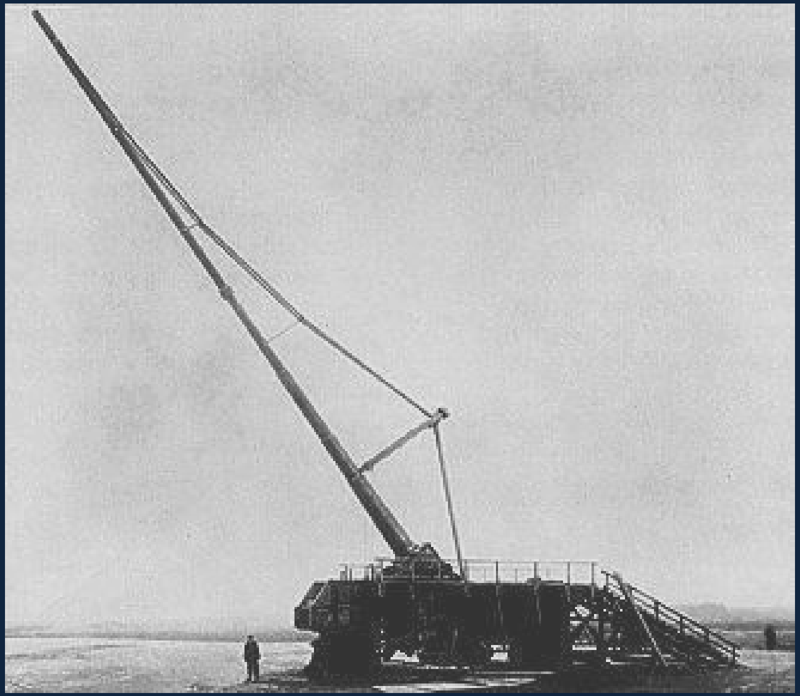
$$\vec{v}_0 = \left(v_{0x}, v_{0y}\right)$$

- The hang time is $t = 2v_{0y} / g$ and during that time the shell travels a horizontal distance

$$R = v_{0x}t = 2v_{0x}v_{0y} / g.$$

- This is the range.

The Paris gun, used by the German army to shell Paris in 1918. Its range was about 80 miles, 130 km.



http://en.wikipedia.org/wiki/File:Parisgun2.jpg

Maximum Range

- Taking the muzzle velocity v_0 as fixed, how do we vary the angle of firing θ to maximize the range?

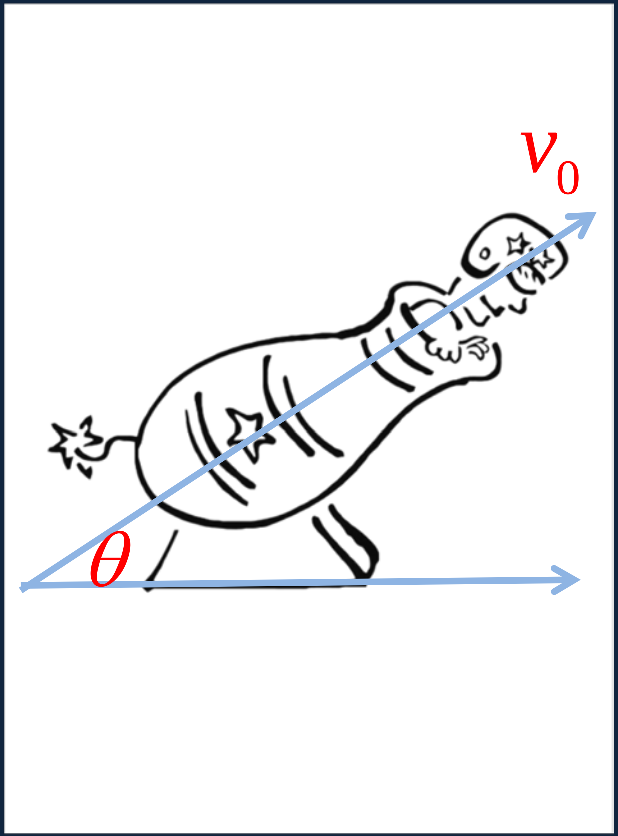
- Now $v_{0x} = v_0 \cos\theta,\ v_{0y} = v_0 \sin\theta$
- So

$$R = 2v_{0x}v_{0y} / g = 2\left(v_0^2 / g\right)\sin\theta\cos\theta$$

$$= \left(v_0^2 / g\right)\sin 2\theta$$

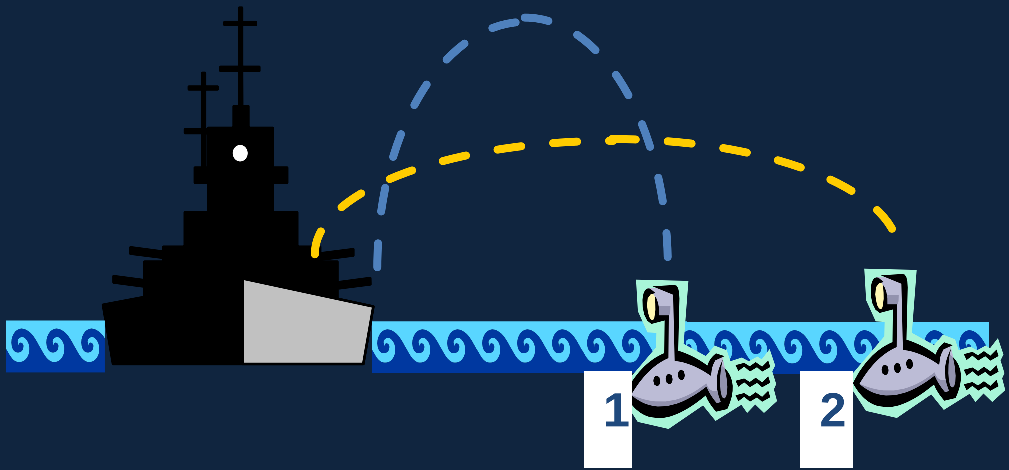
- And maximum range $R_{\max} = v_0^2 / g$ at 45°

http://www.edupics.com/human-cannon-t10746.jpg



ConcepTest 3.7b Punts II

A battleship simultaneously fires two shells at two enemy submarines. The shells are launched with the same initial velocity. If the shells follow the trajectories shown, which submarine gets hit first ?



3) both at the same time

ConcepTest 3.7b Punts II

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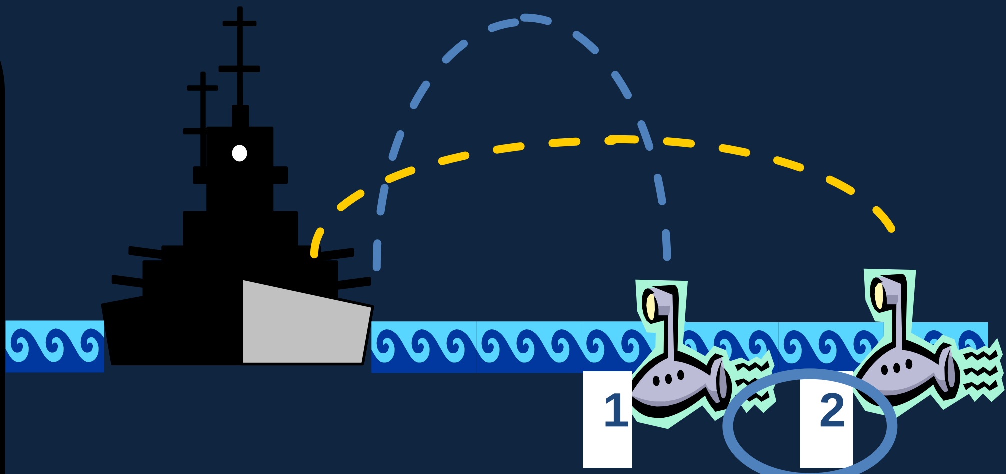
The flight time is fixed by the motion in the *y*-direction. The *higher* an object goes, the *longer* it stays in flight. The shell hitting submarine #2 goes *less high*, therefore it stays in flight for *less time* than the other shell. Thus, submarine #2 is hit first.



3) both at the same time

Follow-up: which one traveled the greater distance?

Problem from Book

- **45.** (II) A high diver leaves the end of a 5.0-m-high diving board and strikes the water 1.3 s later, 3.0 m beyond the end of the board. Considering the diver as a particle, determine (*a*) her initial velocity, (*b*) the maximum height reached; and (*c*) the velocity with which she enters the water.