Hydrodynamics

Physics 1425 Lecture 27

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Basic Concepts

- Fluid conservation
- Bernoulli’s Equation

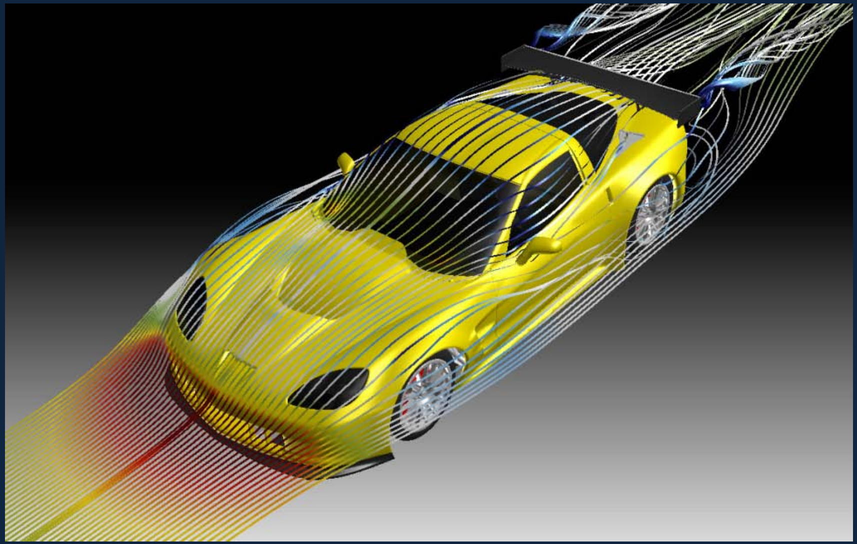
You are sitting in a rowing boat in a small pond. There are some bricks in the boat. You take the bricks and throw them into the pond. They sink to the bottom.

What happens to the water level in the pond, as measured at the bank?

A. It falls.
B. It rises.
C. It stays the same.

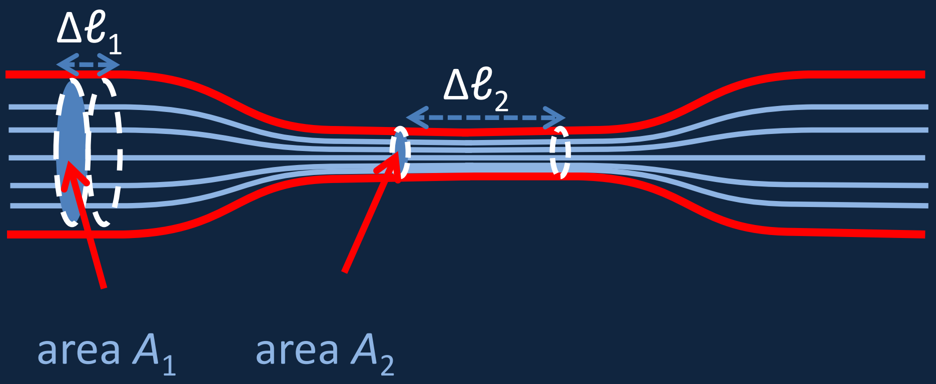
Fluid Flow: Laminar and Turbulent

- In laminar or streamline flow, each particle of fluid follows a smooth path, the streamline.
- Air flow over this Corvette is laminar until the end: the air cannot curve in completely at the back, it breaks away forming a turbulent wake.



Conservation of Fluid

- Suppose fluid is flowing steadily through a pipe which has a narrow section.
- The rate of flow, gallons per sec or cubic meters per sec, must be the same past a point in the narrow part as past a point in the wide part—or fluid will be piling up somewhere!
- So it flows faster through the narrow part.

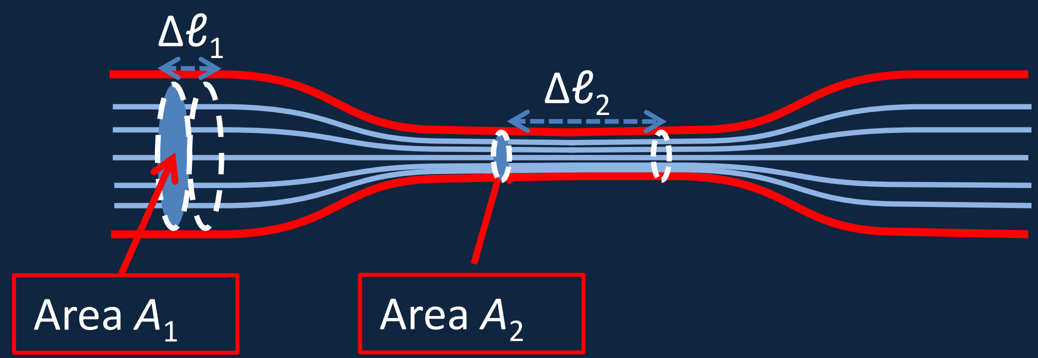


- Imagine a short cylinder of the fluid, of length $\Delta\ell_1$ in the wide part—as it squeezes into the narrow part it gets longer.

- The total mass of fluid Δm in the short cylinder is density x area x length, so

$$\boxed{\Delta m = \rho \Delta V = \rho_1 A_1 \Delta\ell_1 = \rho_2 A_2 \Delta\ell_2}$$

Fluid Velocity: Equation of Continuity

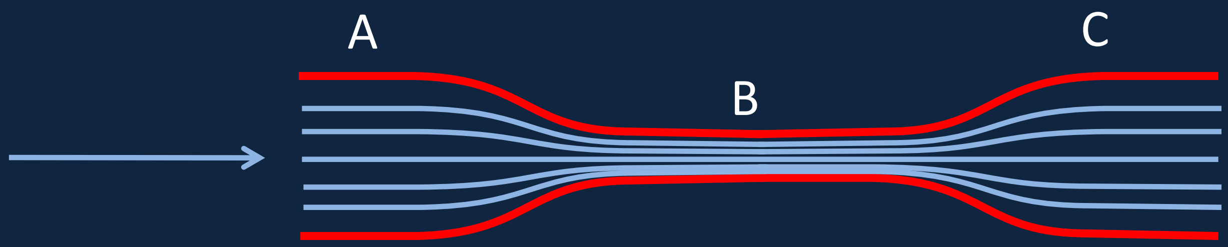


- If the fluid flows distance $\Delta \ell_1$ in the wide tube in time Δt, the mass flow rate past a point is $\Delta m/\Delta t = \rho_1 A_1 \Delta \ell_1 / \Delta t = \rho_1 A_1 v_1$.
- Since the mass flow rate through area A_1 must equal that through A_2 for steady flow,

$$\boxed{\rho_1 A_1 v_1 = \rho_2 A_2 v_2}$$

the "equation of continuity" and often the ρ's can be dropped—water is essentially incompressible, and at low speeds so is air.

Clicker Question



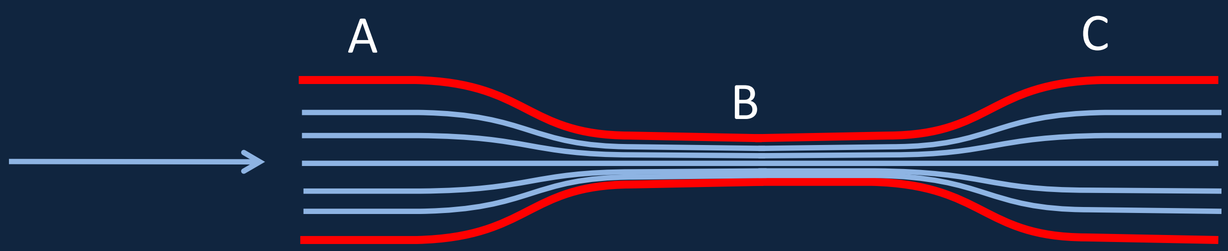
- Where will the pressure be greatest in steady fluid flow?

A. The entering wide part

B. The central narrow part

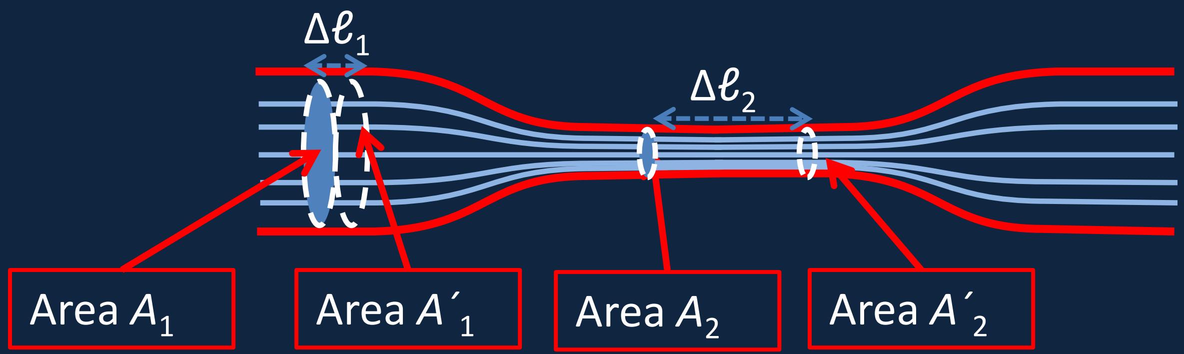
C. The final wide part

Clicker Answer



- Where will the pressure be greatest in steady fluid flow?
- The portion of fluid speeds up as it enters the narrow part—this can only happen if it's pushed from behind. This means *the pressure behind is greater.*
- As it leaves the narrow part, it slows down: again, the pressure is greater in the wide part.

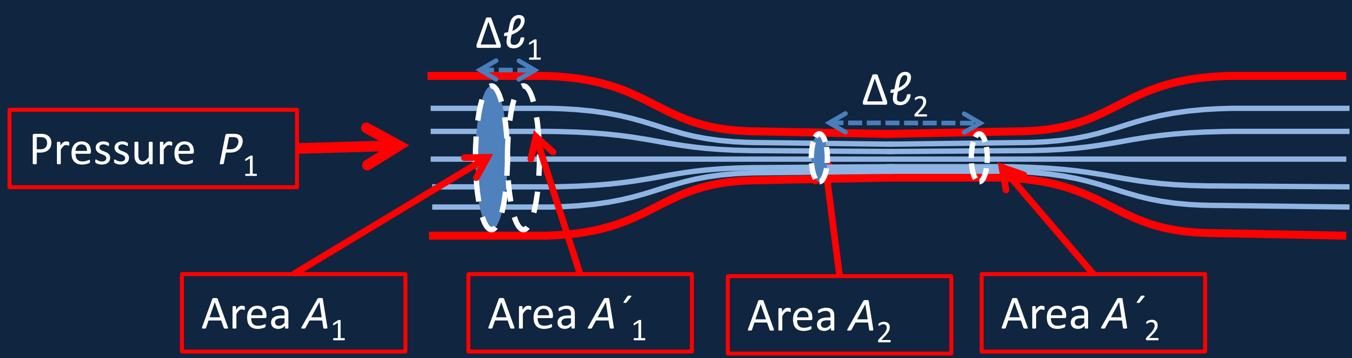
Bernoulli's Equation



[Click here for movie!]

- Focus now on the block of fluid that's between A_1 and A_2 at one instant in time.
- After time Δt, that same fluid will now be between the downstream areas A'_1 and A'_2, and it's picked up some KE!
- A mass $\Delta m = \rho_1 A_1 \Delta\ell_1$ moving at v_1 has been replaced by mass $\rho_2 A_2 \Delta\ell_2$ moving faster—at v_2. From continuity, these masses are the same—so taking ρ constant, there is a KE gain of
- $$\tfrac{1}{2}\Delta m(v_2^2 - v_1^2) = \tfrac{1}{2}\rho A_1 \Delta\ell_1 (v_2^2 - v_1^2).$$

Bernoulli's Equation



For constant density,
$A_1\Delta\ell_1 = A_2\Delta\ell_2$

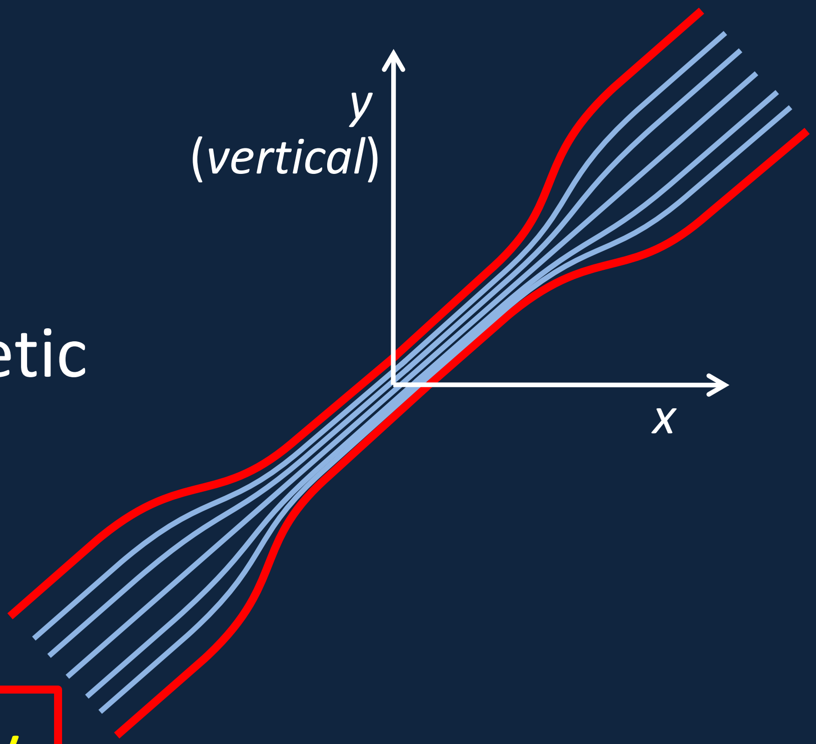
- In the time Δt, there is a KE gain of $\frac{1}{2}\rho A_1 \Delta\ell_1(v_2^2 - v_1^2)$.
- Where did that energy come from?
- In the time Δt, the pressure P_1 on the area A_1 does work:

  force x distance $= P_1 A_1 \Delta\ell_1$
- BUT at the same time, our block of fluid did some work itself: it pushed the fluid in front of it, doing work $= P_2 A_2 \Delta\ell_2$.
- SO net work done $= (P_1 - P_2)\, A_1\Delta\ell_1 =$ KE gain $\frac{1}{2}\rho A_1 \Delta\ell_1(v_2^2 - v_1^2)$
- That is, $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

Uphill Work…

- What if the pipe is *tilted upwards*?
- Now the pressure speeding the fluid along has to lift it as well!
- So the pressure adds potential energy corresponding to how much it was lifted as well as kinetic energy from speeding it up.
- This gives the full Bernoulli's equation:

$$\boxed{P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2}$$



I hold two sheets of paper hanging from my hands parallel, one or two inches apart.
I blow between the two sheets.
What happens?

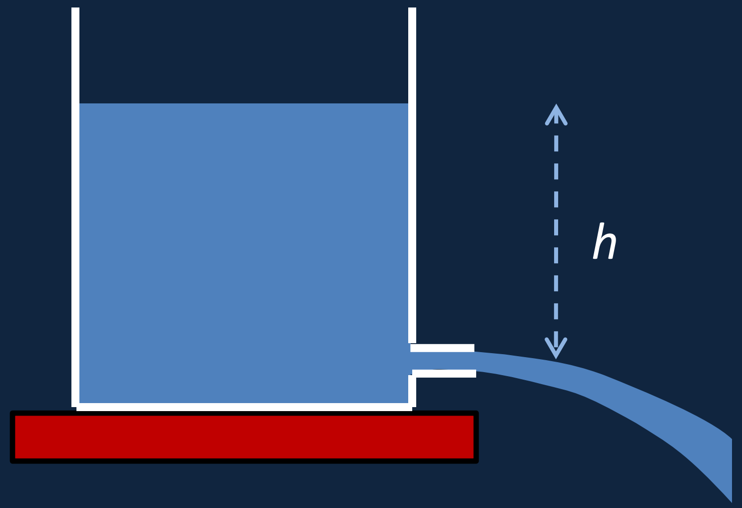
A. They move towards each other.
B. They move apart.

Torricelli's Theorem

- Water coming from a small spigot in a large tank has a speed given by

$$\boxed{v^2 = 2gh}$$

- This is a special case of Bernoulli's equation, because the outside pressure at the spigot is the same as that at the top of the fluid, and fluid velocity at the top is negligible.



ConcepTest 13.15a Fluid Flow

Water flows through a 1-cm diameter pipe connected to a $\frac{1}{2}$-cm diameter pipe. Compared to the speed of the water in the 1-cm pipe, the speed in the $\frac{1}{2}$-cm pipe is:

(1) one-quarter

(2) one-half

(3) the same

(4) double

(5) four times

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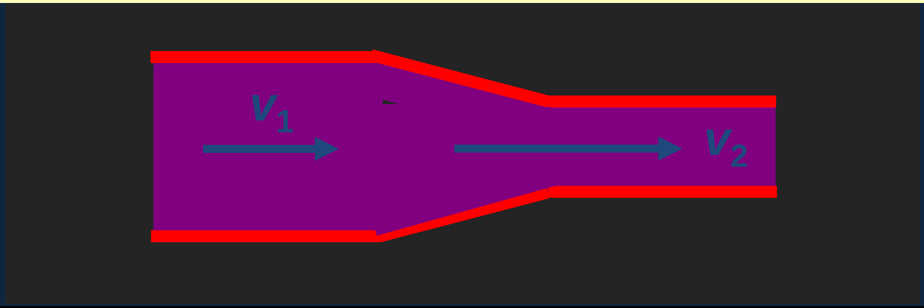
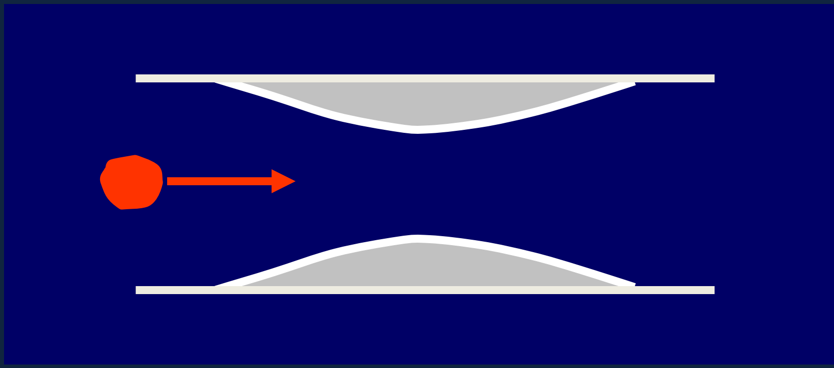


The area of the small pipe is *less*, so we know that the water will flow *faster* there. Because $A \propto r^2$, when the **radius is reduced by one-half,** the **area is reduced by one-quarter,** so the **speed must increase by four times** to keep the flow rate $(A \times v)$ constant.

ConcepTest 13.15b Blood Pressure I

A blood platelet drifts along with the flow of blood through an artery that is partially blocked. As the platelet moves from the wide region into the narrow region, the blood pressure:

1) increases
2) decreases
3) stays the same
4) drops to zero



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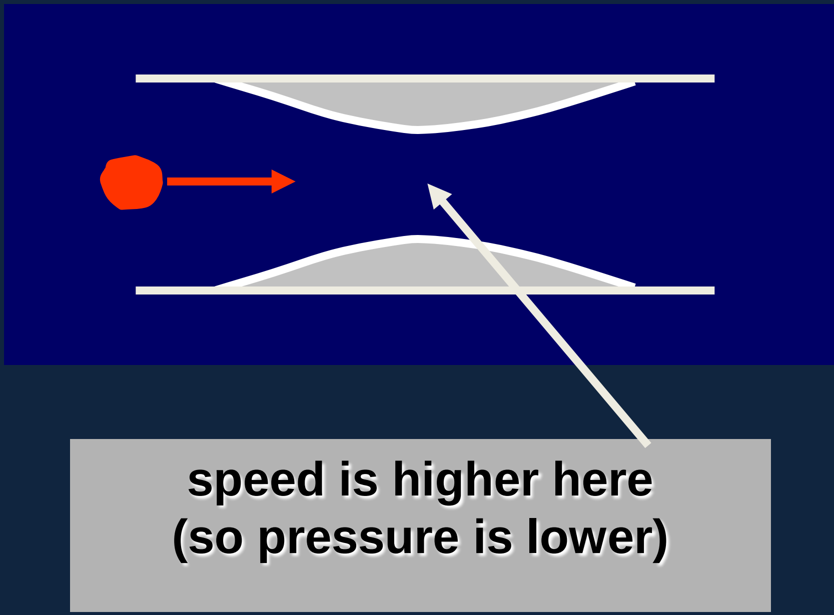
The speed increases in the narrow part, according to the continuity equation. Because the speed is higher, the pressure is lower, from Bernoulli's principle.



ConcepTest 13.15c Blood Pressure II

A person's blood pressure is generally measured on the arm, at approximately the same level as the heart. How would the results differ if the measurement were made on the person's leg instead?

1) **blood pressure would be lower**
2) **blood pressure would not change**
3) **blood pressure would be higher**

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Assuming that the flow speed of the blood does not change, then Bernoulli's equation indicates that at a lower height, the pressure will be greater.