

# More Statics

## Physics 1425 Lecture 24

# Statics: Conditions for Equilibrium

- For any body,  $M d\vec{v}_{\text{CM}} / dt = \sum \vec{F}_i$ , the net force causes the CM to accelerate. Hence, if the body is remaining at rest, 
$$\sum_i \vec{F}_i = 0$$
- To eliminate *angular* acceleration, there must be zero torque about any axis. If all forces are in one plane, it's enough to prove zero torque about one axis perpendicular to the plane:

$$\sum_i \vec{\tau}_i = 0$$

# Free Body Diagrams

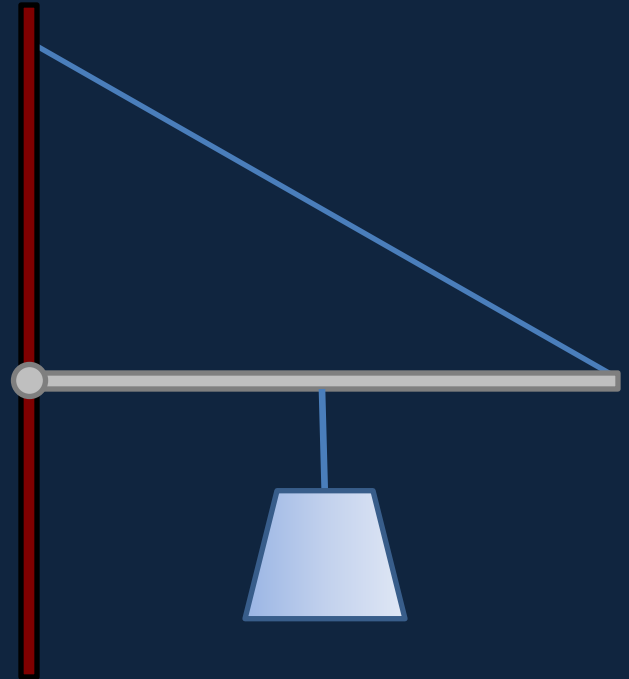
- To apply Newton's Laws to find how a body moves, we must focus on **that body alone** and add **all** the (vector) forces acting on it.
- The diagram showing all the forces on one body (or even part of a body) is called a "**free body diagram**"—we've "freed" the body from the rest of the system, representing everything else just by **the forces on this body**.
- The **net (total) force** then goes into  $\Sigma \vec{F} = m\vec{a}$ .

# Tips about Torques

- In statics, the net torque of the forces on a body is zero about **any** point: choose a point one (or more) forces already pass through!
- The torque at the origin of a force  $\vec{F}$  acting at the point  $\vec{r}$  has magnitude  $rF_{\perp}$ , where  $F_{\perp}$  is the component of the applied force perpendicular to the line from the origin.

# Clicker Question

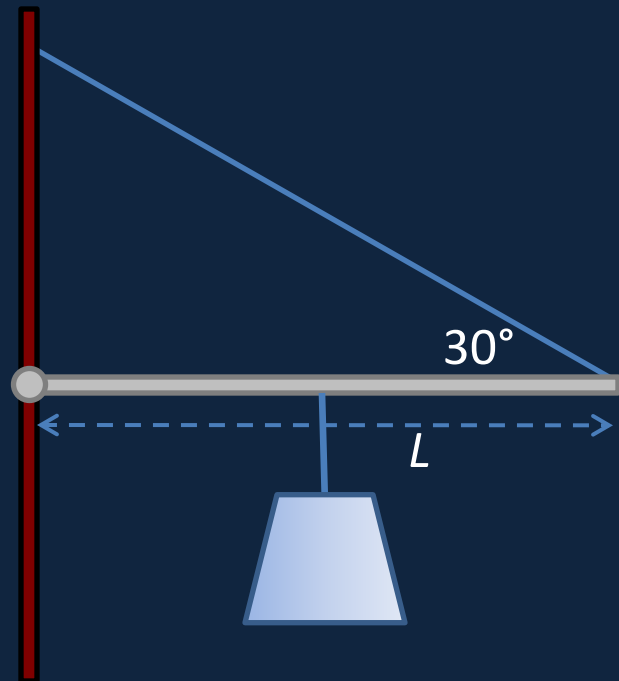
- What is the approx tension  $T$  in the top string, given the mass is 2 kg, and it's hung from the midpoint of the rod, which is light and hinged, the angle is  $30^\circ$ ?
- A. 10 N
- B. 20 N
- C.  $20\sqrt{3}$  N
- D. 40 N



# Clicker Answer

- What is the approx tension  $T$  in the top string, given the mass is 2 kg, and it's hung from the midpoint of the rod, which is light and hinged, the angle is  $30^\circ$ ?

- A. 10 N
- B. 20 N**
- C.  $20\sqrt{3}$  N
- D. 40 N

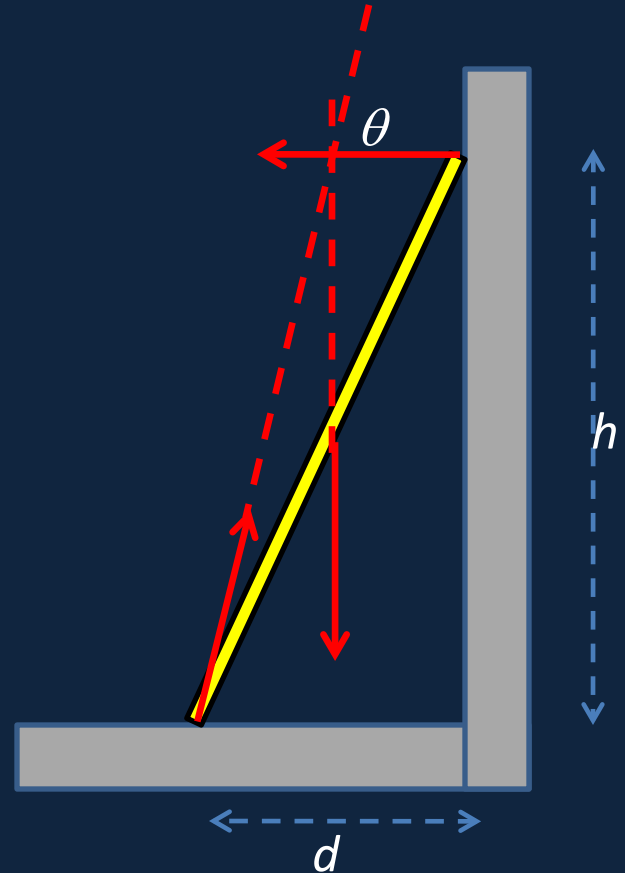


The distance from the hinge to the line of action of the force is  $L\sin 30^\circ = L/2$ .

Alternatively, the component of the tension force perpendicular to the rod is  $T\sin 30^\circ = T/2$ .

# Ladder Against a Smooth Wall

- Assume the wall exerts no significant friction, so the only force from the wall is the normal force.
- What angle does the floor force make with the horizontal?
- To ensure no torque, all three forces' lines of action must pass through the same point.
- Gravity acts at the midpoint, so  $\tan \theta = h/0.5d$ .



**Warning:** this trick saves working with components, but does NOT work if there are more than three forces! (Someone on the ladder.)

# Window Washer's Nightmare?

- Is it possible for a ladder to rest against a wall in equilibrium if the wall is rough but the floor is smooth? (Not counting balanced vertically flat against the wall!).
  - A. Yes
  - B. No



# Window Washer's Nightmare

- Is it possible for a ladder to rest against a wall in equilibrium if the **wall** is **rough** but the **floor** is **smooth**? (Not counting balanced vertically flat against the wall!).
  - A. Yes
  - B. No: because without floor friction, there is no horizontal force to balance the normal force from the wall.
- (You could also note that the floor force and gravity are parallel, so the three forces don't pass through the same point.)

# Hooke's Law

- Hooke (same time as Newton) did many things: here he's holding a cord. (He'd found the shape, called a catenary, inverted, to be the ideal shape for constructing an arch.)
- **His Law:** for most materials under tension, the distance stretched is proportional to the force:

$$F = k\Delta\ell$$



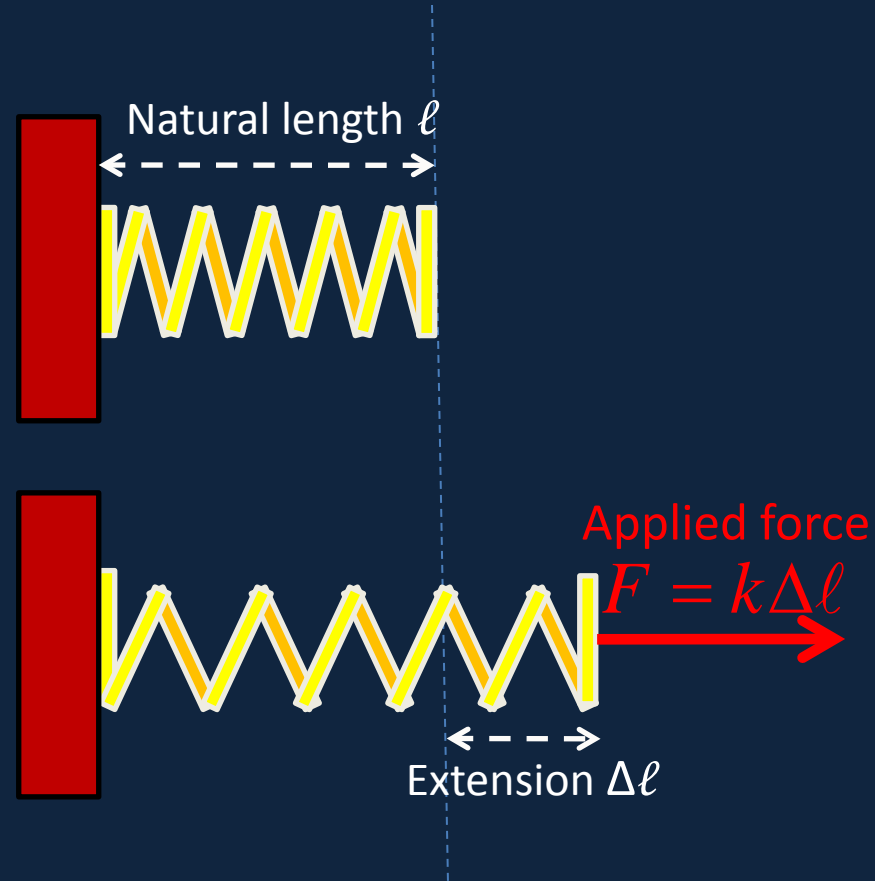
# Review: Force on a Stretched Spring

- To extend a spring beyond its natural length by a distance  $\Delta\ell$  takes a force

$$F = k\Delta\ell$$

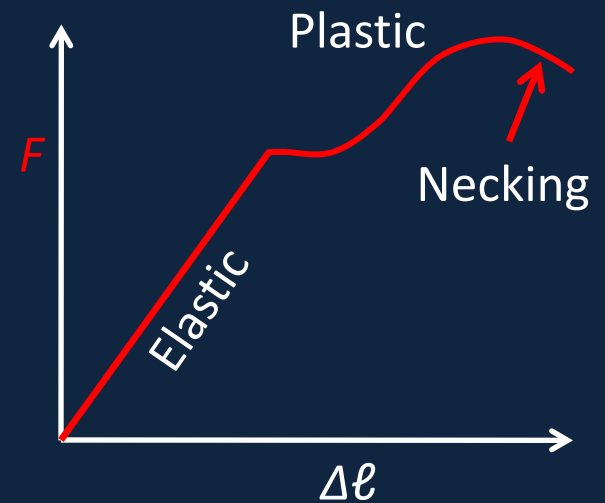
where  $k$  is the “spring constant”.

- This is **Hooke's Law**: and (as we've previously discussed) the spring pulls back with an equal but opposite force.



# Hooke's Law $F = k\Delta\ell$

- Hooke's Law is not a fundamental Law, like Newton's laws: it's a summary of observations on a wide variety of materials.
- But everything has a breaking point—Hooke's Law works fine up to a force not far from the breaking force. Where it works is called the “**elastic region**”, if stretched this far the material **will spring back when released**. Just before breaking, it enters the “**plastic region**”—it stretches **irreversibly**.



The shape of the curve in the plastic region differs from one substance to another—just before breaking, narrowing often occurs at one place : “necking”.

# Young's Modulus

- For a given tension, a wire **twice as long** will stretch **twice** as much—think of it as two wires end to end, under the same tension  $F$ .
- For a given applied force  $F$ , a wire with **twice the cross-section area**  $2A$  will only stretch **half** as much—think of that as two identical parallel wires, each now carrying half the force.
- Young put these facts together to write: 
$$\Delta\ell = \frac{1}{E} \frac{F}{A} \ell_0$$
- The **constant**  $E$  is called **Young's modulus**.