Center of Mass

Physics 1425 Lecture 17

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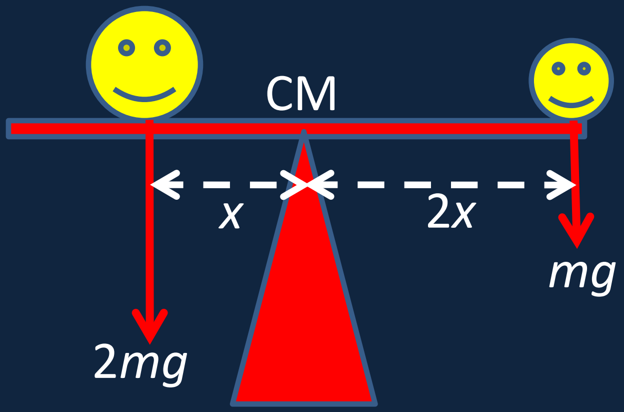
Center of Mass and Center of Gravity

- Everyone knows that if one kid has twice the weight, the other kid must sit twice as far from the axle to balance.
- Each kid then has the same torque about the axle:
- **Torque = force x distance from the axle of the force's line of action.**
- The gravitational forces balance about the axle: it's at the center of gravity—aka the center of mass.



Center of Mass in One Dimension

- Recall the center of mass of two objects is defined by

$$\left(m_1+m_2\right)x_{\mathrm{CM}}=Mx_{\mathrm{CM}}=m_1x_1+m_2x_2$$

- Notice that if we take x_{CM} as the origin (the center of mass frame) then the equation is just

$$m_1x_1+m_2x_2=0$$

precisely the balance equation from before (one of those *x*’s is negative, of course).

CM of Several Objects in One Dimension

- The general formula is:

$$x_{\text{CM}} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i} = \frac{\sum_{i=1}^{n} m_i x_i}{M}$$

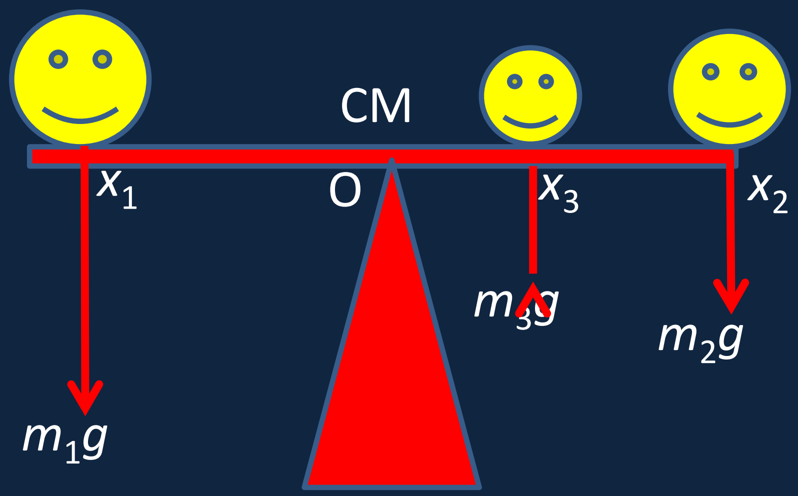
- But before putting in numbers, it's worth staring at the system to see if it's symmetric about any point!

Add Another Kid to the Seesaw…

- For the three to be in balance, the sum of the torques about the axle must be zero, so:

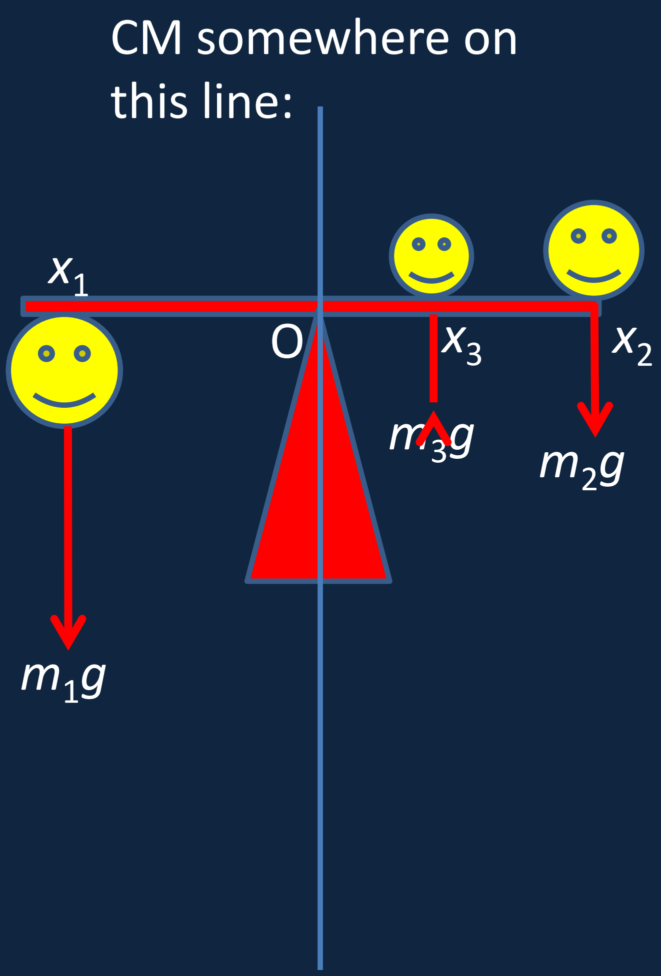
$$m_1 x_1 + m_2 x_2 + m_3 x_3 = 0$$

- That is to say, the *x* coordinate of the center of mass must be the same as the *x*-coordinate of the axle.
- This is clearly extendable to *any* number of masses ….



Some Gymnastics

- The equation
  $$m_1x_1 + m_2x_2 + m_3x_3 = 0$$
  is still correct even if one kid is hanging by his hands below the seesaw!
- The center of mass is not *at* the balance point (the axle) but *is in the same vertical straight line*.



Center of Mass of a Two-Dimensional Object

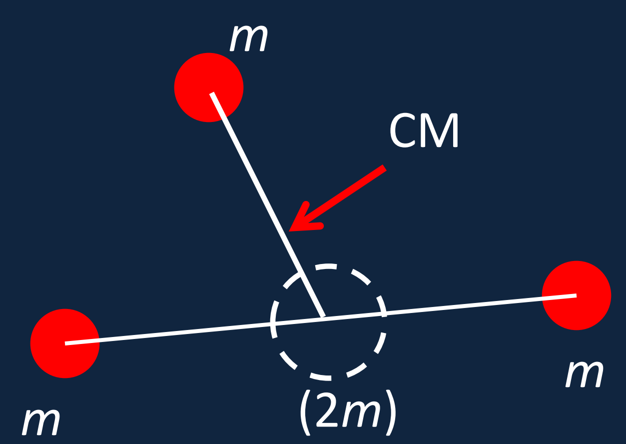
- Think of some shape cut out of cardboard.
- Hang it vertically by pushing a pin through some point.
- Think of it as made up of many small masses—when it's hanging at rest, the center of mass will be somewhere on the vertical line through the pin. Draw the line.
- Repeat with the pin somewhere else: the lines you drew meet at the CM.

Tip: if the object is symmetric about some line, the center of mass will be *on that line*!

Three Equal Masses

- If we have three equal masses at the corners of a triangle, the center of mass of two of them is the half-way point on the side joining them.
- We can replace them by a mass 2*m* at that point, then the CM of *all three masses* is on the line from the other vertex to that point, one-third of the way up.
- This is the centroid of the triangle, and is at

$$\vec{r} = \frac{\vec{r}_1 + \vec{r}_2 + \vec{r}_3}{3}$$

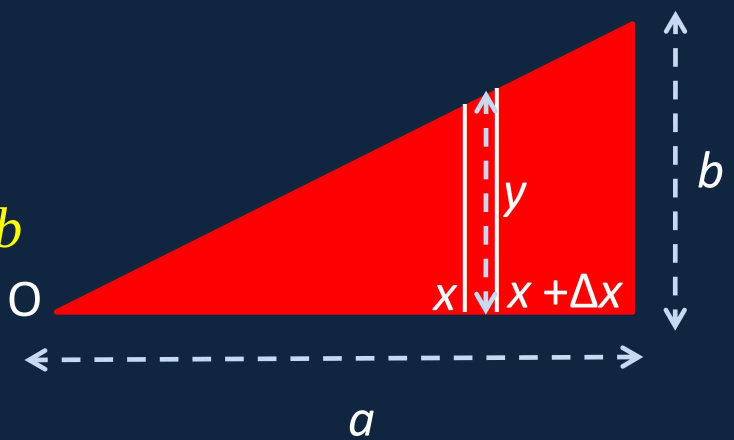


Center of Mass of a Solid Triangle

- We'll take a right-angled triangle. The *x*-coordinate of the CM is found by the integral generalization of the sum $$Mx_{\mathrm{CM}} = \sum_{i=1}^{n} m_i x_i$$
- If the triangle has area mass density ρ kg/m^2, the strip shown has mass $\rho y \Delta x$, and $M = \frac{1}{2}\rho ab$, so

$$\tfrac{1}{2}\rho ab x_{\mathrm{CM}} = \int_0^a \rho xy\,dx = (b/a)\int_0^a \rho x^2 dx = \tfrac{1}{3}\rho a^2 b$$

from which the CM is at $(2/3)a$.



- Bottom line: the CM of the solid triangle is at the same point as the CM of three equal masses at the corners!

The height y of the strip at x is given by $y/b = x/a$, from similar triangles.

ConcepTest 9.10a Elastic Collisions I

Consider two elastic collisions:

1) a golf ball with speed *v* hits a stationary bowling ball head-on.

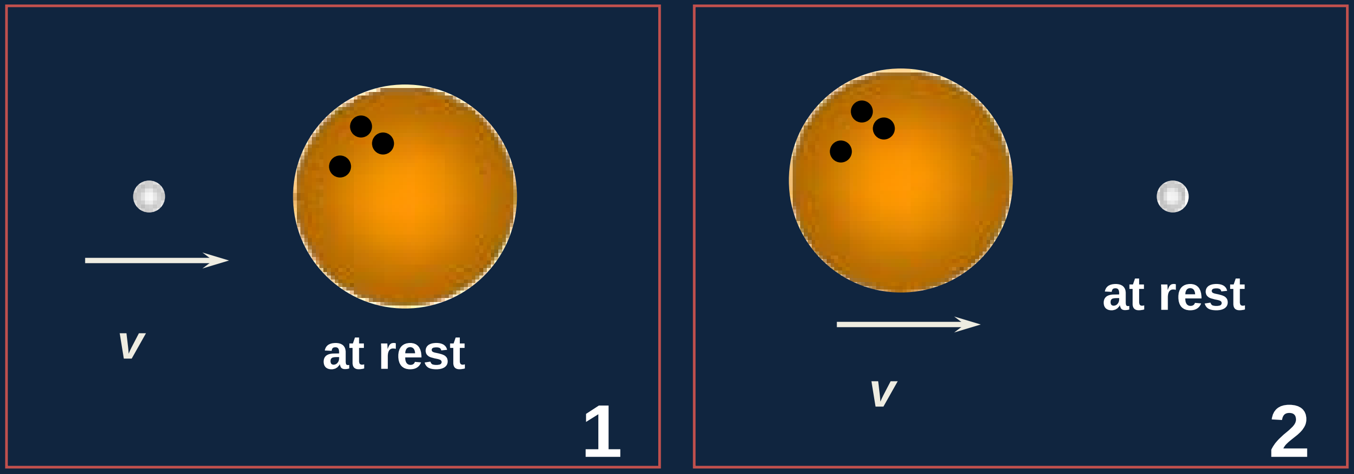
2) a bowling ball with speed *v* hits a stationary golf ball head-on.

In which case does the golf ball have the greater speed after the collision?

1) situation 1

2) situation 2

3) both the same



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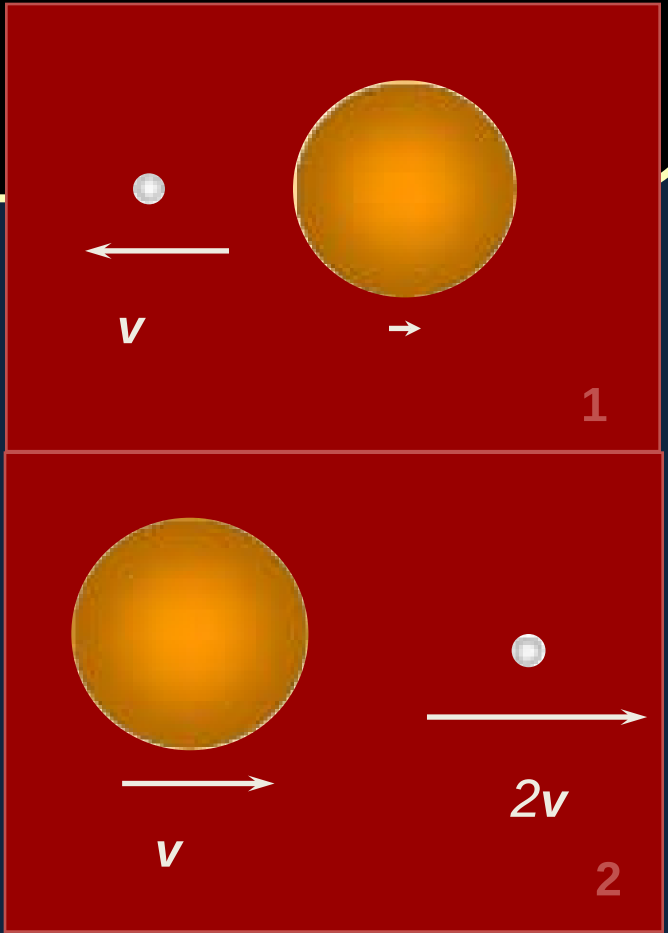

Remember that the magnitude of the *relative velocity* has to be equal before and after the collision!

In case **1** the bowling ball will almost remain at rest, and the **golf ball** will **bounce back with speed close to v**.

In case **2** the bowling ball will keep going with speed close to v, hence the **golf ball** will **rebound with speed close to $2v$**.

ConcepTest 9.11 Golf Anyone?

You tee up a golf ball and drive it down the fairway. Assume that the collision of the golf club and ball is elastic. When the ball leaves the tee, how does its speed compare to the speed of the golf club?

1) **greater than**
2) **less than**
3) **equal to**

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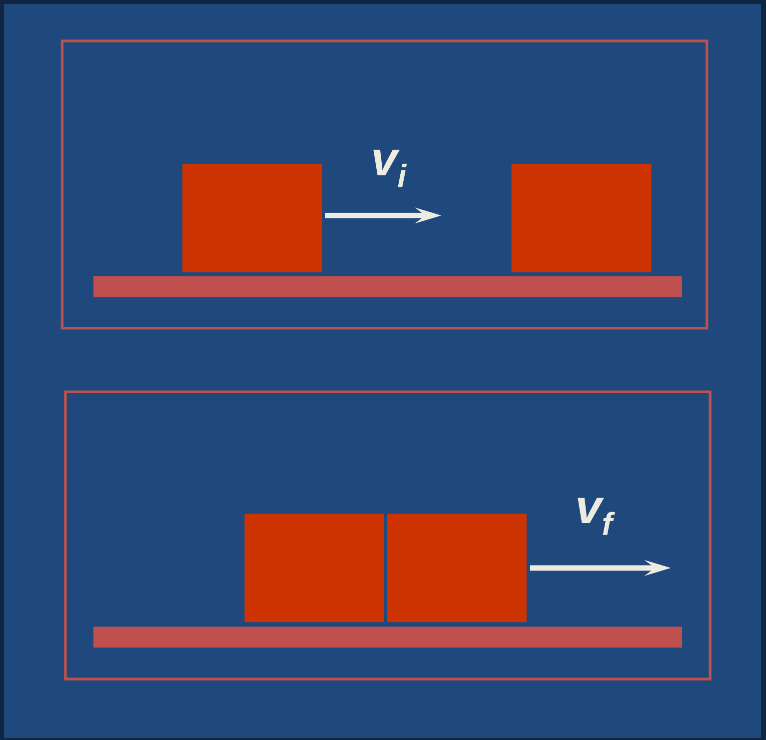
If the speed of approach (for the golf club and ball) is v, then the speed of recession must also be v. Because the golf club is hardly affected by the collision and it continues with speed v, then the ball must fly off with a speed of $2v$.

ConcepTest 9.12b Inelastic Collisions II

On a frictionless surface, a sliding box collides and sticks to a second identical box that is initially at rest. What is the final KE of the system in terms of the initial KE?

1) $KE_f = KE_i$
2) $KE_f = KE_i / 4$
3) $KE_f = KE_i / \sqrt{2}$
4) $KE_f = KE_i / 2$
5) $KE_f = \sqrt{2}\, KE_i$



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Momentum: $mv_i + 0 = (2m)v_f$

So we see that: $v_f = \frac{1}{2} v_i$

Now, look at kinetic energy:

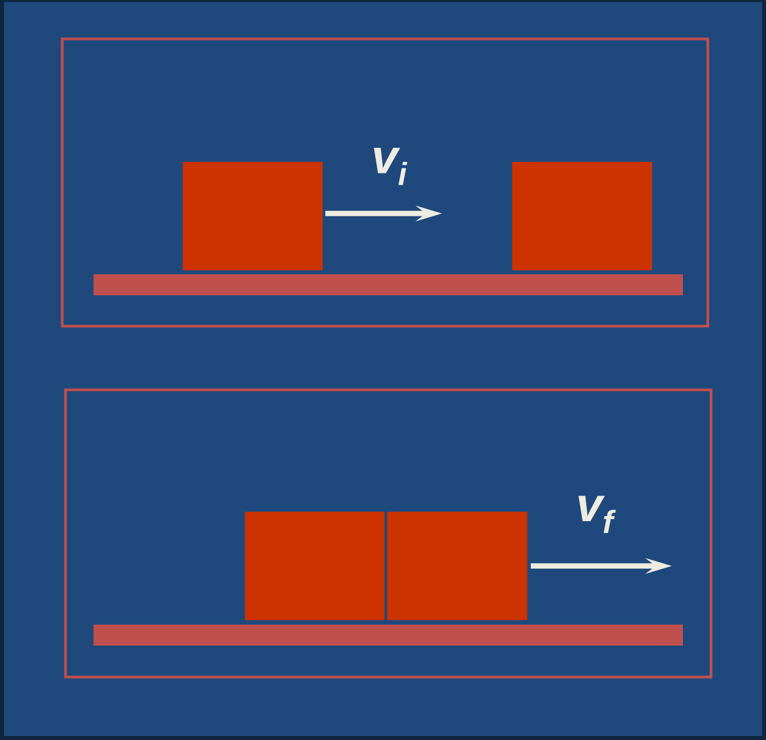
First, $KE_i = \frac{1}{2} m v_i^2$

So: $KE_f = \frac{1}{2} m_f v_f^2$

$= \frac{1}{2} (2m)(1/2\, v_i)^2$

$= \frac{1}{2} (1/2\, m v_i^2)$

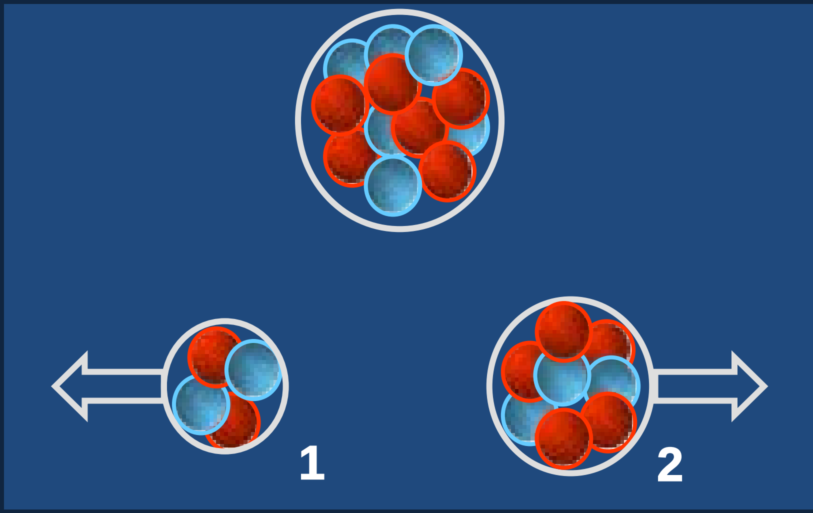
$= \frac{1}{2} KE_i$



ConcepTest 9.13a Nuclear Fission I

A uranium nucleus (at rest) undergoes fission and splits into two fragments, one heavy and the other light. Which fragment has the greater momentum?

1) the heavy one
2) the light one
3) both have the same momentum
4) impossible to say



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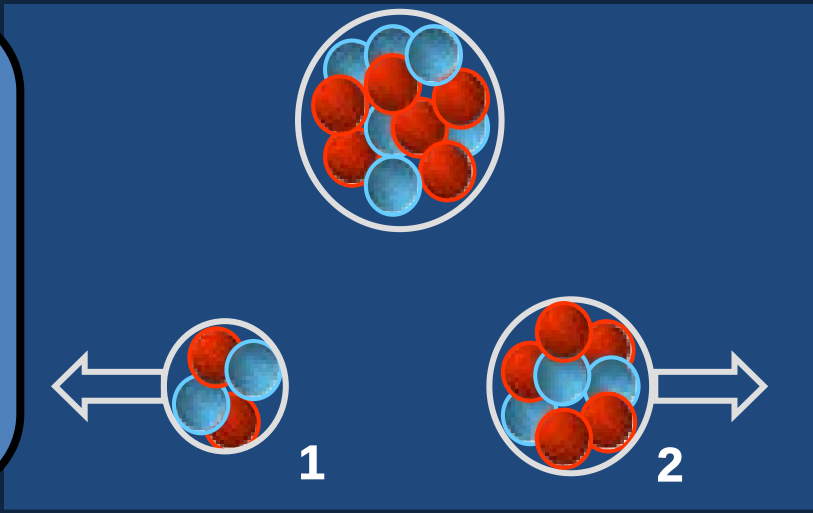
The initial momentum of the uranium was zero, so the final total momentum of the two fragments must also be zero. **Thus the individual momenta are equal in magnitude and opposite in direction.**



ConcepTest 9.13b Nuclear Fission II

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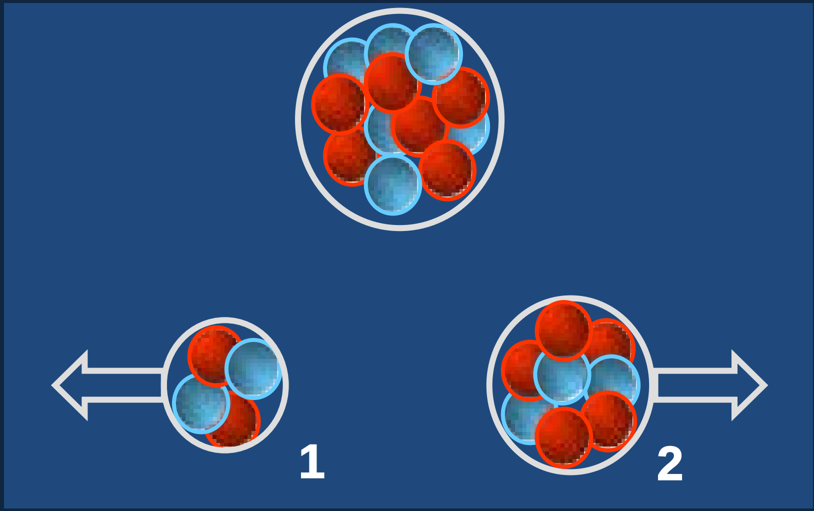

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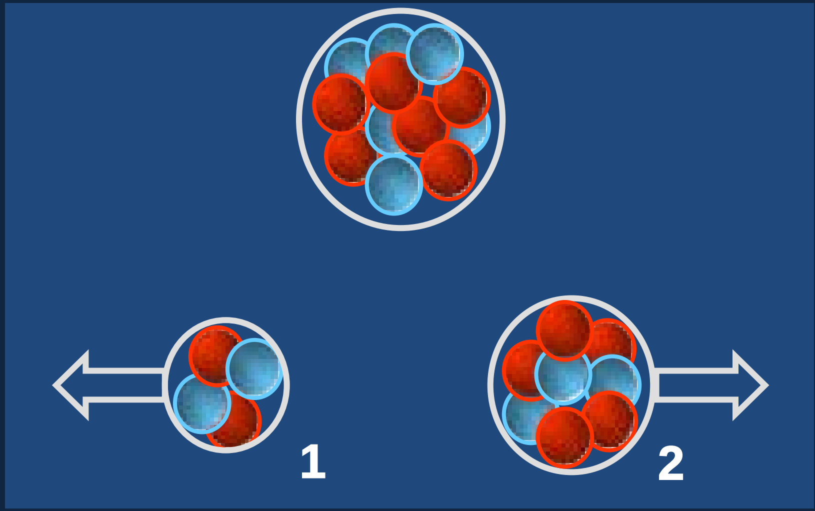
$MV = mv$, $M > m$ so v > V, therefore

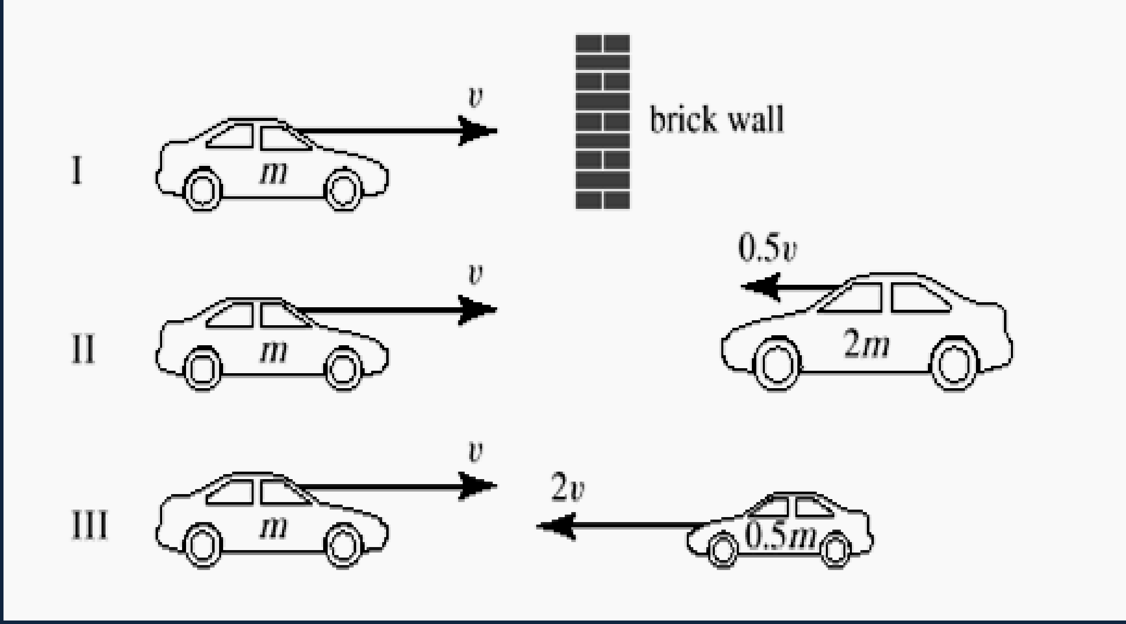
$\frac{1}{2} MV^2 < \frac{1}{2} mv^2$.



ConcepTest 9.16a Crash Cars I

If all three collisions below are totally inelastic, which one(s) will bring the car on the left to a complete halt?

1) I
2) II
3) I and II
4) II and III
5) all three



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3) I and II
4) II and III
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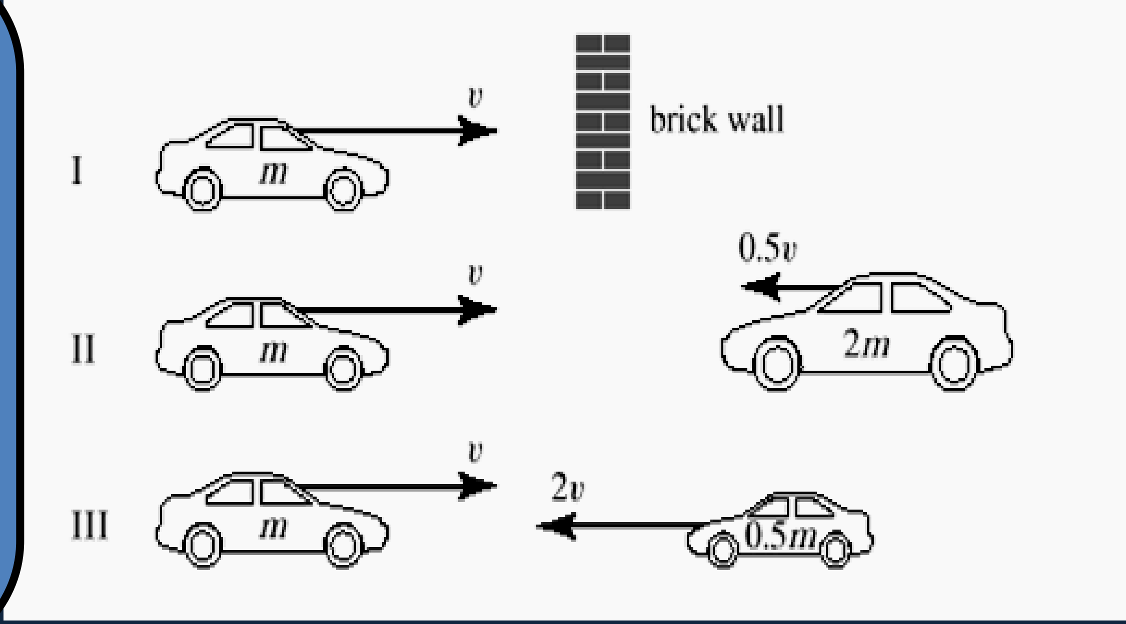
In case I, the solid wall clearly stops the car.

In cases II and III, because **p_{tot} = 0 before the collision**, then **p_{tot} must also be zero after the collision**, which means that the car comes to a halt in all three cases.



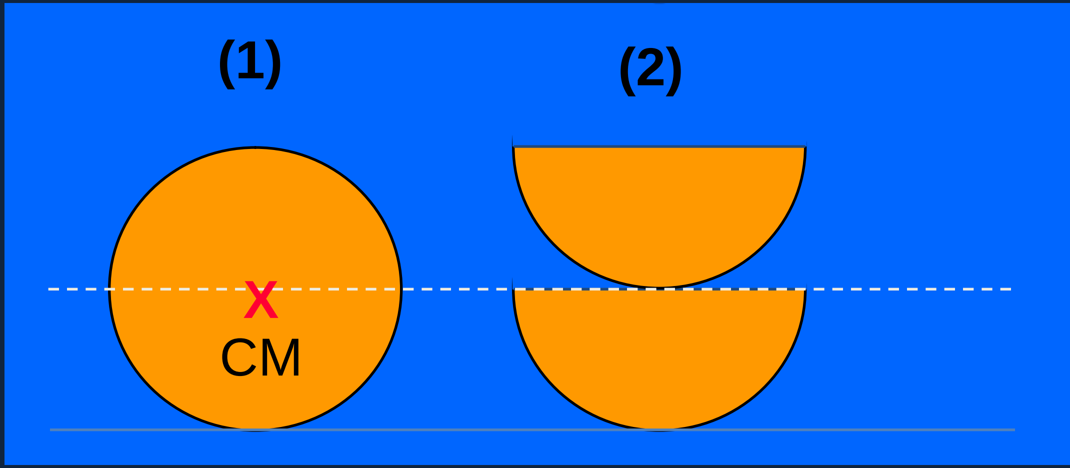
ConcepTest 9.20 Center of Mass

The disk shown below in (1) clearly has its center of mass at the center.

Suppose the disk is cut in half and the pieces arranged as shown in (2).

Where is the center of mass of (2) as compared to (1) ?

1) higher
2) lower
3) at the same place
4) there is no definable CM in this case



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The CM of each half is closer to the top of the semicircle than the bottom. The CM of the whole system is located at the **midpoint of the two semicircle CMs**, which is **higher** than the yellow line.