Kinetic Energy and Energy Conservation

Physics 1425 Lecture 13

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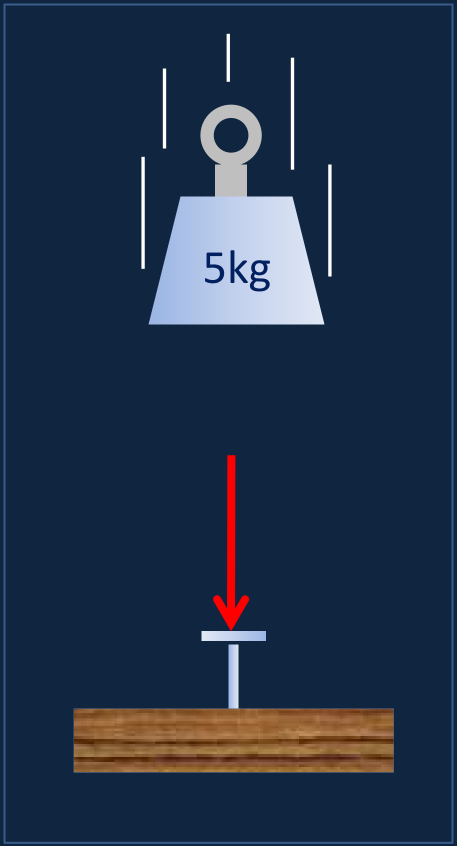
Moving Things Have Energy

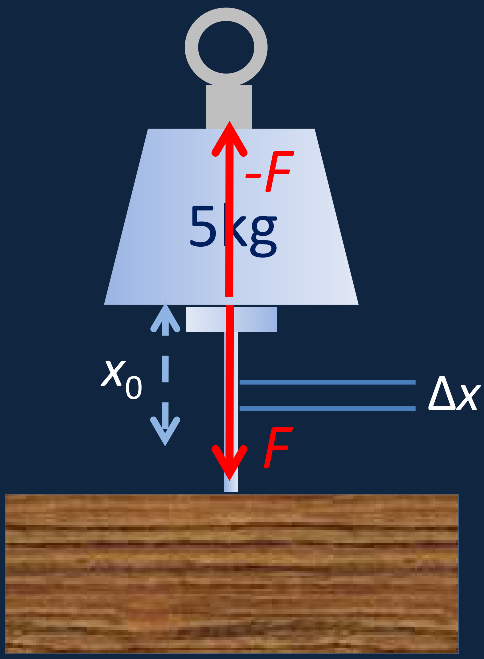
- Energy is the ability to do work: to deliver a force that acts through a distance.
- Placing a weight gently on a nail does nothing.
- Dropping the weight on the nail can drive the nail into the wood.
- If the weight is moving when it hits the nail, it has the ability to do work driving the nail in. This is its *kinetic energy.*



How Much Work Does the Moving Weight Do?

- After contact with the nail, the forces between the weight and the nail are equal and opposite. Suppose the nail is driven in a total distance x_0.
- In going through a small distance Δx, the work done on the nail $\Delta W = F\Delta x$.
- Meanwhile, for the weight $-F = ma$, the weight has slowed down: $-F = m\Delta v/\Delta t$.
- Therefore $\Delta W = F\Delta x = -m\, \Delta v\Delta x/\Delta t$.
- Now for small Δx, we can take $\Delta x/\Delta t = v$, so $\Delta W = -mv\Delta v$, and

$$W = \int_0^{x_0} dW = -\int_{v_0}^{0} mv\,dv = \tfrac{1}{2} mv_0^2$$



Weight hits nail with speed v_0

Where Did the Weight's Energy Come From?

- We've seen that if the weight hits the nail and comes rapidly to rest, it loses energy $\frac{1}{2} mv_0^2$.
- This is its kinetic energy K at speed v_0.
- Let's suppose it gained that energy by being dropped from rest at a height h.
- At uniform acceleration g, $v_0^2 = 2gh$.
- So the kinetic energy $\frac{1}{2} mv_0^2 = mgh$: precisely the potential energy lost in the fall—the work done by gravity mgh = force mg x distance h.

ConcepTest 7.5b Kinetic Energy II

Car #1 has twice the mass of car #2, but they both have the same kinetic energy. How do their speeds compare?

1) $2v_1 = v_2$
2) $\sqrt{2}v_1 = v_2$
3) $4v_1 = v_2$
4) $v_1 = v_2$
5) $8v_1 = v_2$

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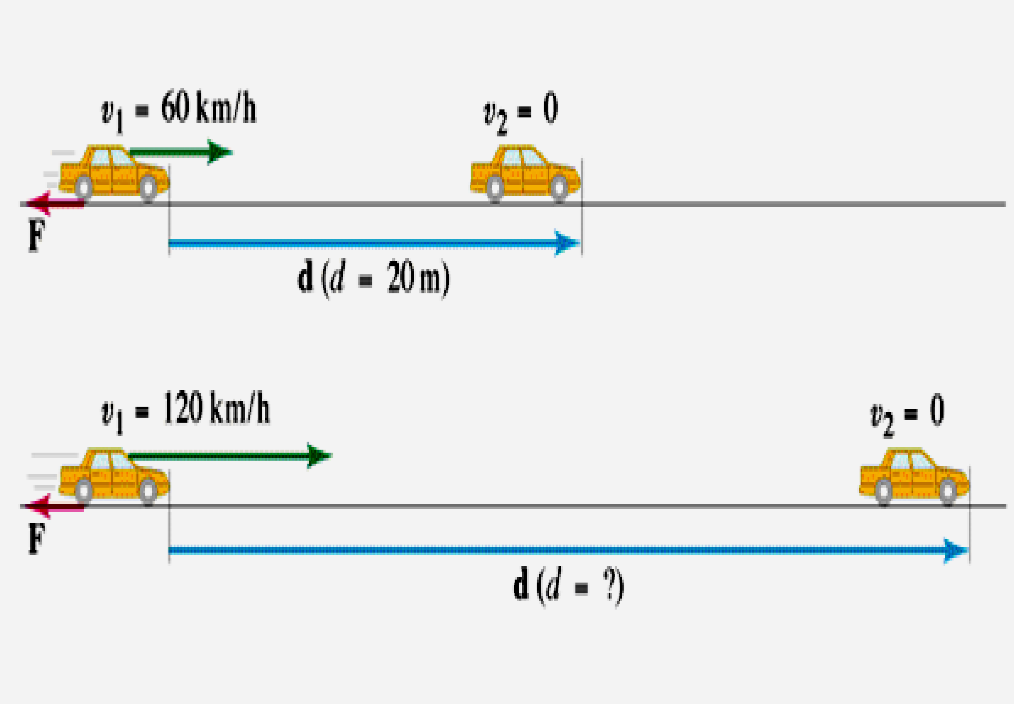
Because the kinetic energy is $\frac{1}{2}mv^2$, and the mass of car #1 is greater, then car #2 must be moving faster. If the ratio of m_1/m_2 is 2, then the ratio of v^2 values must also be 2. This means that the ratio of v_2/v_1 must be the square root of 2.

ConcepTest 7.8a Slowing Down

If a car traveling 60 km/hr can brake to a stop within 20 m, what is its stopping distance if it is traveling 120 km/hr? Assume that the braking force is the same in both cases.

1) 20 m
2) 30 m
3) 40 m
4) 60 m
5) 80 m



ConcepTest 7.8a Slowing Down

If a car traveling **60 km/hr** can brake to a stop within **20 m**, what is its stopping distance if it is traveling **120 km/hr**? Assume that the braking force is the same in both cases.

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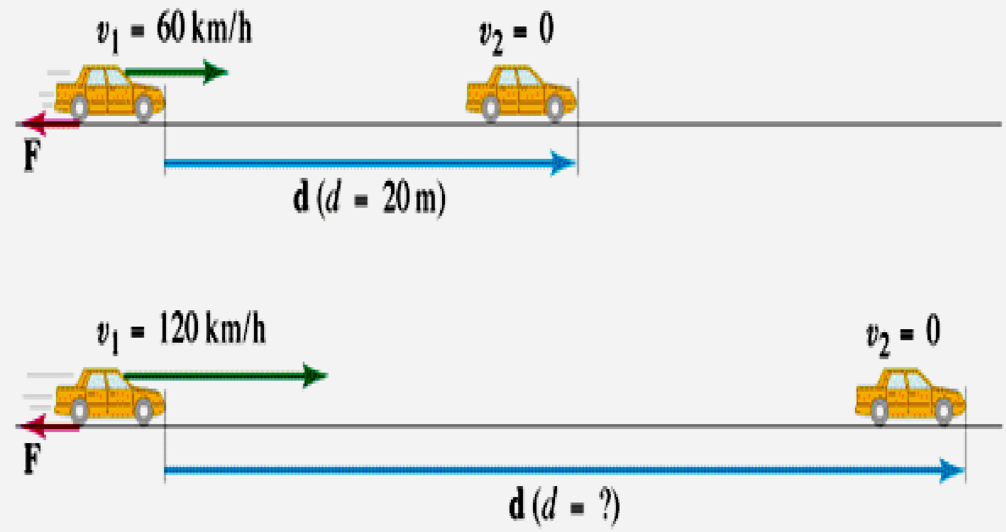
$F\,d = W_{net} = \Delta KE = 0 - \frac{1}{2} mv^2,$

and thus, $|F|\,d = \frac{1}{2} mv^2.$

Therefore, if the speed **doubles**, the stopping distance gets **four times larger**.



ConcepTest 7.8c Speeding Up II

The work W_0 accelerates a car from 0 to 50 km/hr. How much work is needed to accelerate the car from 50 km/hr to 150 km/hr?

1) 2 W_0
2) 3 W_0
3) 6 W_0
4) 8 W_0
5) 9 W_0

ConcepTest 7.8c Speeding Up II

The work W_0 accelerates a car from 0 to 50 km/hr. How much work is needed to accelerate the car from 50 km/hr to 150 km/hr?

1) $2\,W_0$
2) $3\,W_0$
3) $6\,W_0$
4) $8\,W_0$
5) $9\,W_0$

Let's call the two speeds *v* and 3*v*, for simplicity.

We know that the work is given by $W = \Delta KE = KE_f - Ke_i$.

Case #1: $W_0 = \frac{1}{2} m\,(v^2 - 0^2) = \frac{1}{2} m\,(v^2)$

Case #2: $W = \frac{1}{2} m\,((3v)^2 - v^2) = \frac{1}{2} m\,(9v^2 - v^2) = \frac{1}{2} m\,(8v^2) = 8\,W_0$

Follow-up: How much work is required to stop the 150-km/hr car?

A Small Kinetic Energy Change

- Suppose the velocity of a mass *m* changes by a tiny amount $\Delta\vec{v}$ as the mass moves through $\Delta\vec{r}$. Then the change in kinetic energy *K* is (dropping the *very* tiny $(\Delta v)^2$ term)

$$\Delta K = \tfrac{1}{2}m\left(\vec{v}+\Delta\vec{v}\right)^2 - \tfrac{1}{2}m\vec{v}^{\,2} = m\vec{v}\cdot\Delta\vec{v} = m\frac{\Delta\vec{r}\cdot\Delta\vec{v}}{\Delta t}$$

- Note this depends only on the change in *speed*—the dot product ensures that only the component of $\Delta\vec{v}$ in the direction of $\vec{v}$ counts. The displacement $\Delta\vec{r}$ is of course in direction $\vec{v}$.

Energy Balance for a Projectile

- Consider a projectile acted on only by gravity, moving a distance $\Delta\vec{r}$ in a short time Δt.
- Gravity does work $m\vec{g}\cdot\Delta\vec{r} = -\Delta U$, where U is the gravitational potential energy.
- The change in velocity $\Delta\vec{v} = \vec{g}\Delta t$, so the change in potential energy

$$\Delta U = -m\vec{g}\cdot\Delta\vec{r} = -m\frac{\Delta\vec{v}\cdot\Delta\vec{r}}{\Delta t} = -\Delta K\,!$$

- The total energy $U + K$ does not change.

Conservation of Mechanical Energy

- We've established that for a projectile acted on only by gravity K.E. + P.E. = a constant,

$$\tfrac{1}{2}m\vec{v}^2 + mgh = E,$$

- Here *E* is called the total (mechanical) energy.
- This is valid if:

A. We can neglect air resistance, friction, etc.

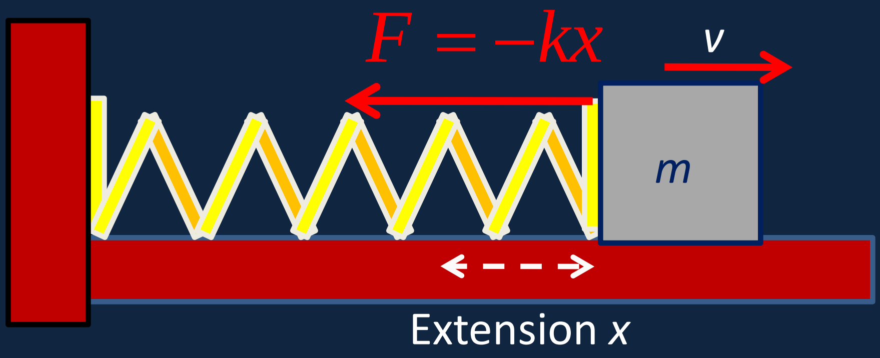
B. Other forces acting are always perpendicular to the direction of motion: so this will also be true for a roller coaster, ignoring friction.

Springs Conserve Energy, Too

- Suppose the spring is fixed to the wall, at the other end a mass *m* slides on a frictionless surface.
- By an exactly similar argument to that for gravity, we can show

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = E,$$

constant total energy.



Conservative and Nonconservative Forces

- Gravity and the spring are examples of conservative forces: if work is done against them, they store it all as potential energy, and it can be used later. Total mechanical energy is conserved.
- Friction is not a conservative force: work done against friction generates heat, it does not conserve the mechanical energy, little of which can be recovered.

Different Paths for a Conservative Force

- For a conservative force, suppose taking an object from point A to point B along path P_1 requires us to supply work W_1. Then if we let the object slide back from B to A, the force will fully reimburse us, giving back *all* the work W_1.
- Now suppose there's another path P_2 from A to B, and using *that* path takes less work from us, W_2.
- We can construct a track going from A to B along P_2 then back along P_1, and we'll gain energy! This is a perpetual motion machine…so what's wrong?

Potential Energy in a Conservative Field

- Imagine a complicated conservative field, like gravity from Earth + Moon at any point. We've established that the work we need to do to take a mass *m* from point $\vec{r}_A$ to $\vec{r}_B$,

$$W\left(\vec{r}_A,\vec{r}_B\right)=\int_{\vec{r}_A}^{\vec{r}_B}\vec{F}\cdot d\vec{r}$$

depends *only* on the endpoints, **not the path**—so we can unambiguously define a potential energy difference

$$U\left(\vec{r}_B\right)-U\left(\vec{r}_A\right)=\int_{\vec{r}_A}^{\vec{r}_B}\vec{F}\cdot d\vec{r}$$

Potential Energy Determines Force

- If we know the potential energy $U(\vec{r})$ in a complicated gravitational field, how can we find the gravitational force on a mass *m* at $\vec{r}$?
- Take a very short path going in the *x*-direction:

$$U(\vec{r}+\Delta x)-U(\vec{r})=\int_{\vec{r}}^{\vec{r}+\Delta x}\vec{F}\cdot d\vec{r}=F_x\Delta x$$

- We must apply a force F_x to move this small distance, so the opposing gravitational force is given by $F_{Gx}=-\partial U(\vec{r})/\partial x$.

More on Potential Energy and Force

- Since the potential energy is given by integrating the force through a distance, it's not surprising that we get back the force by differentiating the potential energy.

- For gravity near the Earth's surface, $U(\vec{r}) = mgz$, taking *z* as vertically up, so

$$F_z = -\partial U / \partial z = -mg$$

  and since *U* doesn't depend on *x* or *y*, there is no force in those directions.

- **Reminder!** Forces and work depend only on *changes* in potential energy—we can set the zero of potential energy wherever is convenient, like ground level.

Potential Energy and Force for a Spring

- For a spring,

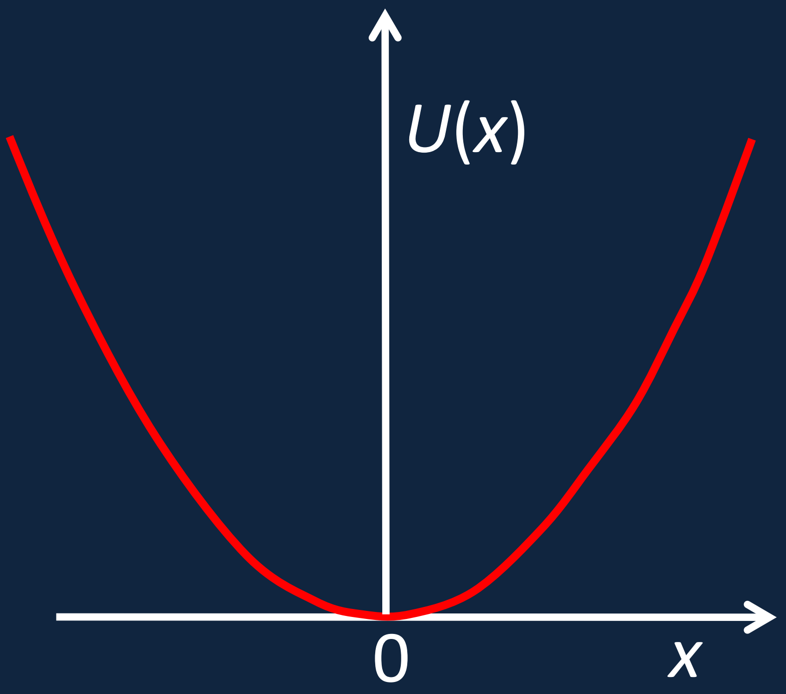
  $$U(x) = \tfrac{1}{2}kx^2$$

  a parabola.

The force the spring exerts when extended to *x*

$$F(x) = -dU(x)/dx = -kx$$

- It's worth staring at the *U*(*x*) graph, bearing in mind that the force at any point is the *negative* of the slope there—see how it gets steeper further away from the origin.



ConcepTest 8.1 Sign of the Energy II

Is it possible for the gravitational potential energy of an object to be negative?

1) yes
2) no

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Gravitational PE is *mgh*, where height *h* is measured relative to some arbitrary reference level where PE = 0. For example, a book on a table has positive PE if the zero reference level is chosen to be the floor. However, if the ceiling is the zero level, then the book has negative PE on the table. Only *differences* (or changes) in PE have any physical meaning.

ConcepTest 8.2 KE and PE

You and your friend both solve a problem involving a skier going down a slope, starting from rest. The two of you have chosen **different levels for $y = 0$** in this problem. Which of the following quantities will you and your friend agree on?

1) only B
2) only C
3) A, B, and C
4) only A and C
5) only B and C

A) skier's PE

B) skier's change in PE

C) skier's final KE

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A) skier's PE   B) skier's change in PE   C) skier's final KE

The **gravitational PE depends upon the reference level**, but the ***difference*** **ΔPE does not**! The work done by gravity must be the same in the two solutions, so **ΔPE and ΔKE should be the same**.

Follow-up: Does anything change *physically* by the choice of $y = 0$?

ConcepTest 8.5 Springs and Gravity

A mass attached to a vertical spring causes the spring to stretch and the mass to move downwards. What can you say about the spring's potential energy (PE_s) and the gravitational potential energy (PE_g) of the mass?

1) both PE_s and PE_g decrease
2) PE_s increases and PE_g decreases
3) both PE_s and PE_g increase
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The spring is stretched, so its elastic PE increases, because $PE_s = \frac{1}{2} kx^2$. The mass moves down to a lower position, so its gravitational PE decreases, because $PE_g = mgh$.

Problem from Book

- **15.** (II) A 50-kg bungee jumper leaps from a bridge. She is tied to a bungee cord that is 10 m long when unstretched, and falls a total of 30 m. (*a*) Calculate the spring constant *k* of the bungee cord assuming Hooke's law applies. (*b*) Calculate the maximum acceleration she experiences.