Work and Energy

Physics 1425 Lecture 12

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What is Work and What Isn't?

- In physics, work has a very restricted meaning!
- *Doing homework isn't work.*

- Carrying somebody a mile on a level road isn't work…

- Lifting a stick of butter three feet *is* work—in fact, about one unit of work.

Work is *only* done by a *force*…

- and, the force has to move something!
- Suppose I lift one kilogram up one meter…
- I do it at a slow steady speed—my force just balances its weight, let's say 10 Newtons.

- **Definition:** *if I push with 1 Newton through 1 meter, I do work 1 Joule.*

- So lifting that kilogram took 10 Joules of work.

Only motion *in the direction of the force* counts …

- Carrying the weight straight across the room at constant height does no work on the weight.
- After all, it could have been just sliding across on ice—and the ice does no work!

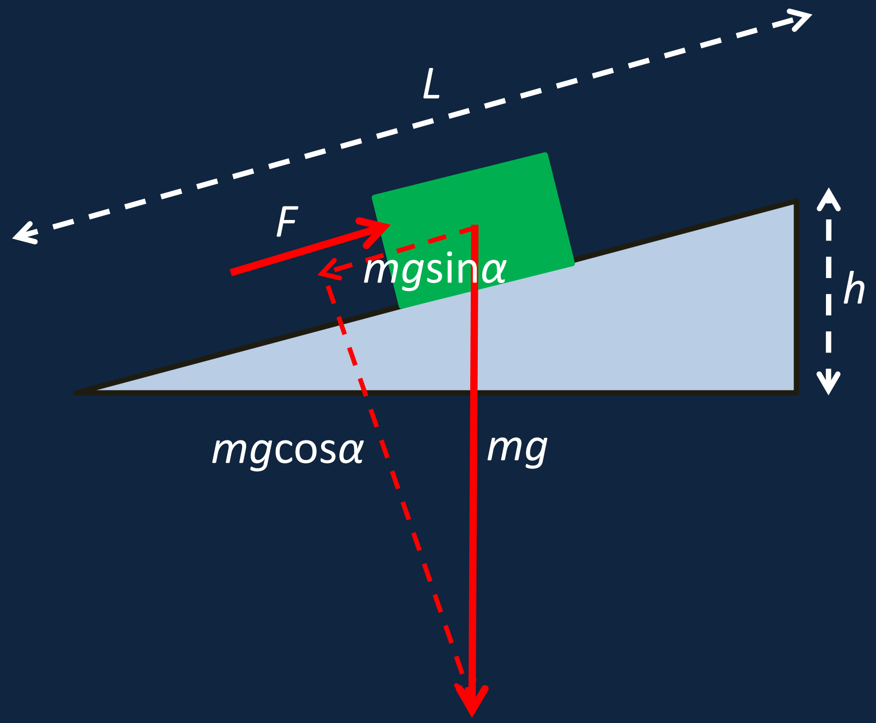
- What about pushing a box at constant velocity up a frictionless slope?

Pushing a box up a frictionless slope…

- Suppose we push a box of mass m at a steady speed a distance L up a frictionless slope of angle α.
- The work done is

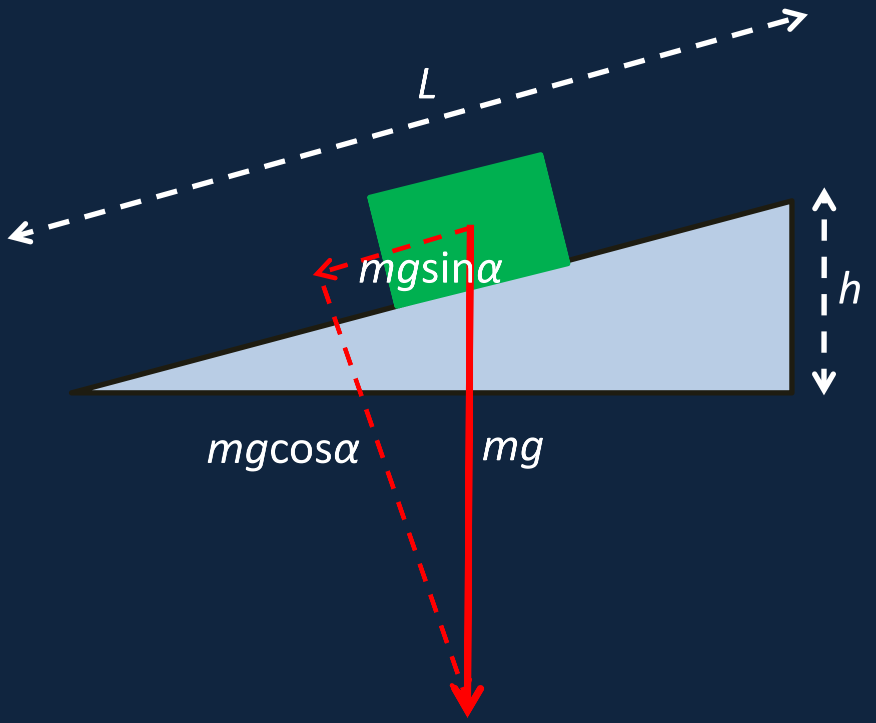
  $FL = mgL\sin\alpha = mgh$

  where h is the height gained.
- Meanwhile, gravity is doing *negative* work… its force is directed *opposite to the motion*.



…and letting it slide back down.

- Letting the box go at the top, the force of gravity along the slope, *mgL*sin*α,* will do exactly as much work on the box on the way down as we did pushing it up.
- Evidently, the work we did raising the box was *stored* by gravity.
- This "stored work" is called **potential energy** and is written $U = mgh$



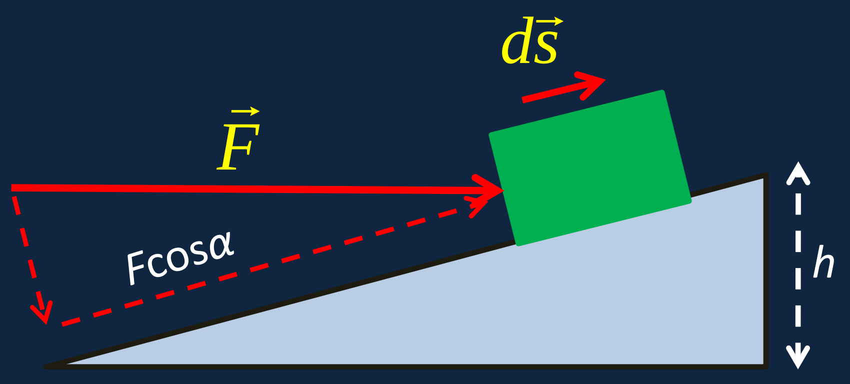
Energy is the Ability to Do Work

- We've established that pushing the box up a frictionless slope against gravity stores—in gravity—the ability to do work on the box on its way back down.
- This "stored work" is called potential energy.
- Notice it depends *not* on the slope, but only on the net height gained:

$$U = mgh.$$

What if you push the box *horizontally*?

- The box only moves up the slope, so only the component of force in that direction does any work.
- If the box moves a small distance *ds*, the work done
- $dW = (F\cos\alpha)ds$.
- This vector combination comes up a lot: we give it a special name… $dW = \vec{F} \cdot d\vec{s}$



The Vector Dot Product $\vec{A} \cdot \vec{B}$

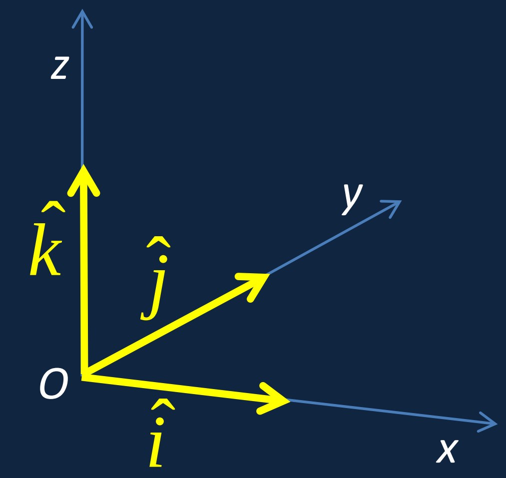
- The dot product of two vectors is defined by:

  $$\vec{A} \cdot \vec{B} = AB\cos\theta$$

  where *A*, *B* are the lengths of the vectors, and θ is the angle between them.
- Alternately: The dot product is the length of $\vec{A}$ multiplied by the length of the component of $\vec{B}$ in the direction of $\vec{A}$.
- From this $\vec{A} \cdot \left(\vec{B} + \vec{C}\right) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$.
- If the vectors are perpendicular, $\vec{A} \cdot \vec{B} = 0$.

Dot Product in Components

- Recall we introduced three orthogonal unit vectors $\hat{i}, \hat{j}, \hat{k}$ pointing in the directions of the *x*, *y* and *z* axes respectively.



- Note $\hat{i} \cdot \hat{i} = \hat{i}^2 = 1, \ \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0.$

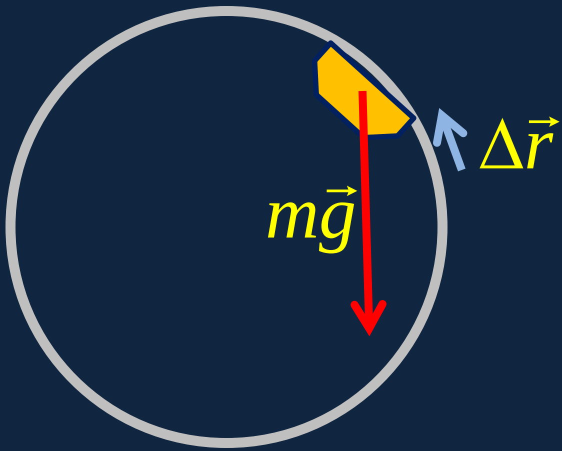
- Writing $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ we find

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z.$$

Positive and Negative Work

- As the loop the loop car climbs a small distance $\Delta\vec{r}$, the force of gravity $m\vec{g}$ does work $\vec{F}\cdot\vec{d} = m\vec{g}\cdot\Delta\vec{r}$. This is negative on the way up—the angle between the two vectors is more than 90°.
- Total work around part of the loop can be written

$$W = \sum \vec{F}\cdot\Delta\vec{r} = \int_{\vec{a}}^{\vec{b}} \vec{F}\cdot d\vec{r}$$



ConcepTest 7.2c Play Ball!

In a baseball game, the catcher stops a 90-mph pitch. What can you say about the work done by the catcher on the ball?

1) **catcher has done positive work**
2) **catcher has done negative work**
3) **catcher has done zero work**

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The force exerted by the catcher is **opposite in direction to the displacement of the ball, so the work is negative**. Or using the definition of work ($W = F\,d\cos\theta$), because $\theta = 180^\circ$, then $W < 0$. Note that because the work done on the ball is negative, its speed decreases.

Follow-up: What about the work done by the ball on the catcher?

ConcepTest 7.2d Tension and Work

A ball tied to a string is being whirled around in a circle. What can you say about the work done by tension?

1) tension does no work at all
2) tension does negative work
3) tension does positive work

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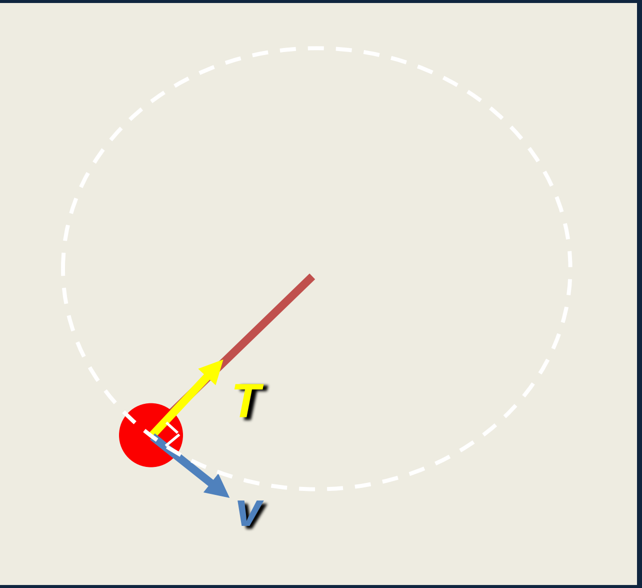
No work is done because the force acts in a perpendicular direction to the displacement. Or using the definition of work ($W = F\,d\cos\theta$), because $\theta = 90$, then $W = 0$.

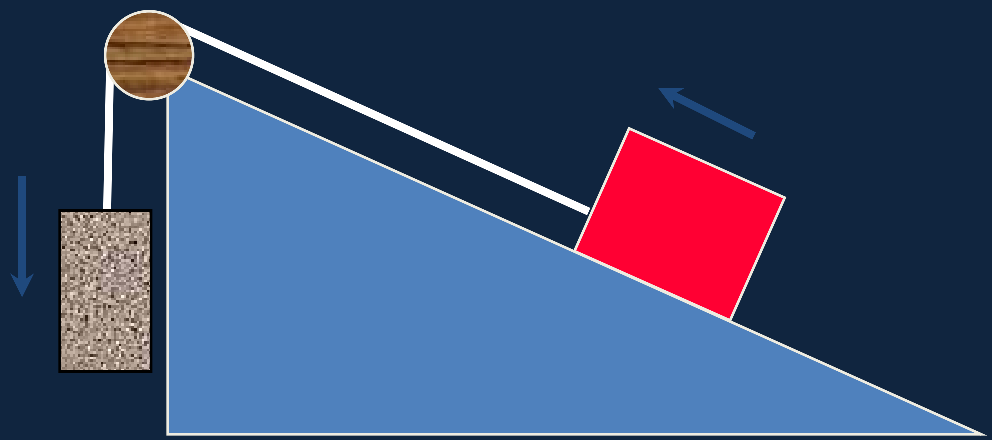


Follow-up: Is there a force in the direction of the velocity?

ConcepTest 7.3 Force and Work

A box is being pulled up a rough incline by a rope connected to a pulley. How many forces are doing work on the box?

1) one force
2) two forces
3) three forces
4) four forces
5) no forces are doing work



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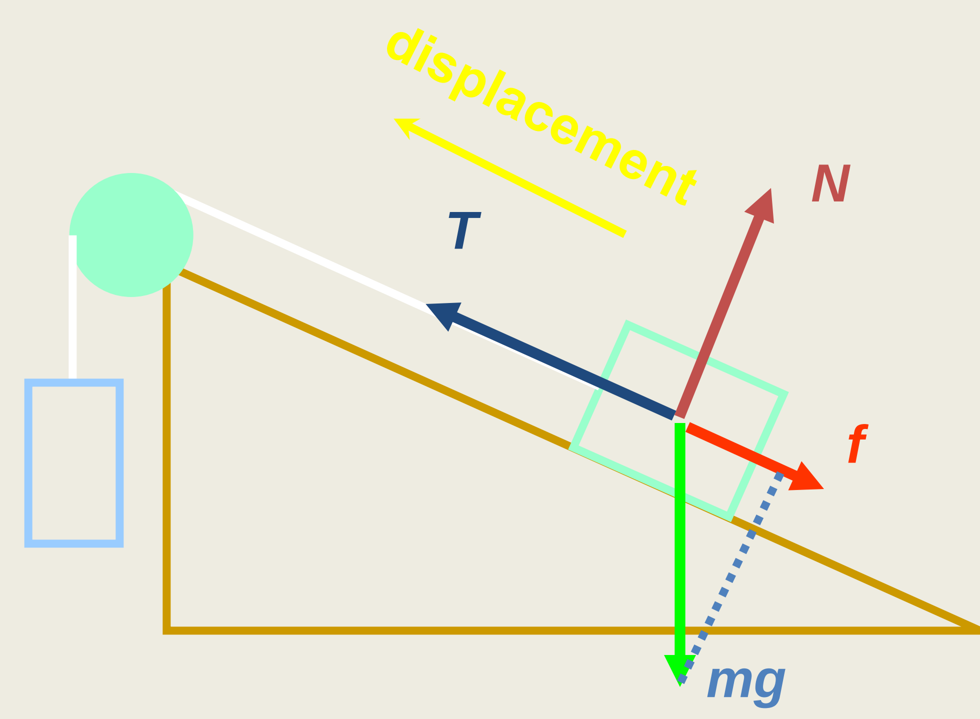
Any force not perpendicular to the motion will do work:

N does ***no work***

T does **positive** work

f does negative work

mg does negative work



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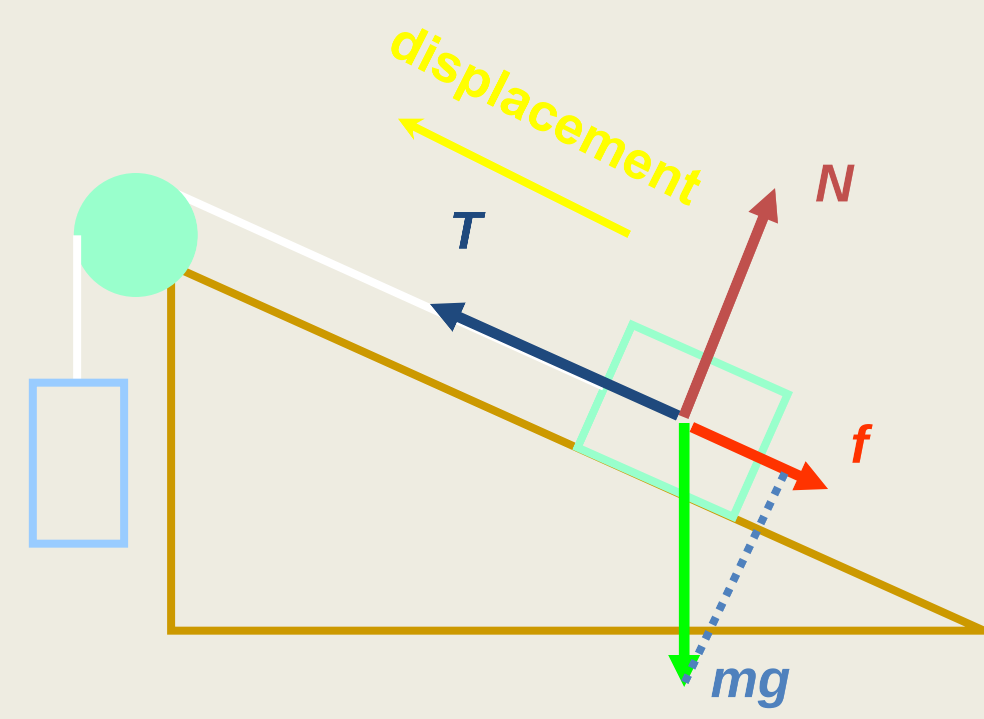
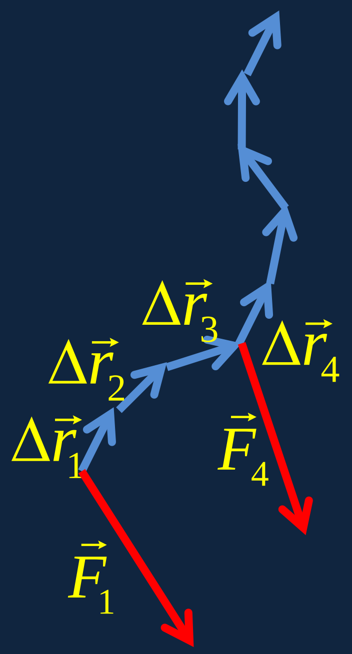


Work done by any Force along any Path

- The expression for work done along a path is general: just break the path into small pieces, add the work for each piece, then go to the limit of tinier pieces to give an integral:

$$W = \sum_i \vec{F}_i \cdot \Delta\vec{r}_i = \int_{\vec{a}}^{\vec{b}} \vec{F} \cdot d\vec{r}$$



In components:

$$W = \int_{a_x}^{b_x} F_x dx + \int_{a_y}^{b_y} F_y dy + \int_{a_z}^{b_z} F_z dz$$

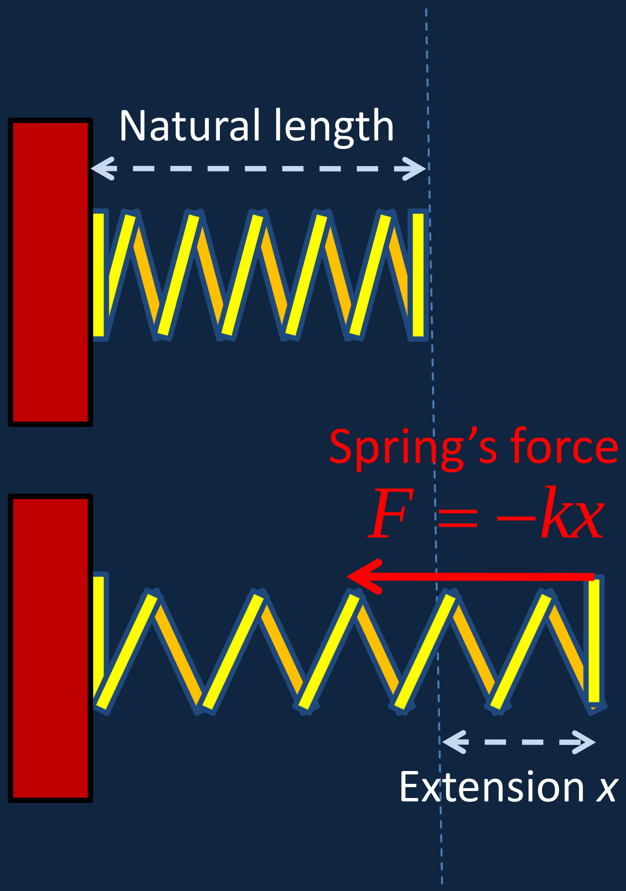
Force of a Stretched Spring

- If a spring is pulled to extend beyond its natural length by a distance *x*, it will pull back with a force

$$F = -kx$$

where *k* is called the "spring constant".

The same linear force is also generated when the spring is *compressed*.



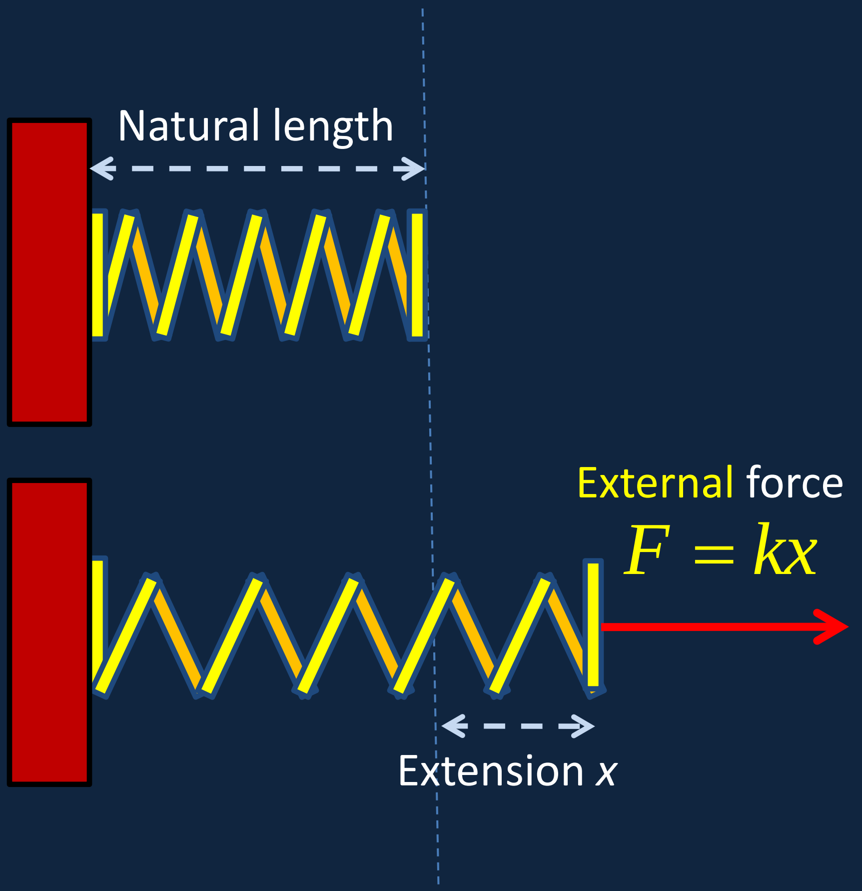
Work done in *Stretching* a Spring

- The work from an external force needed to stretch the spring from

  x to $x+\Delta x$ is $kx\Delta x$,

  so the total work to stretch from the natural length to an extension x_0

  $$W = \int_0^{x_0} kx\,dx = \tfrac{1}{2}kx_0^2.$$

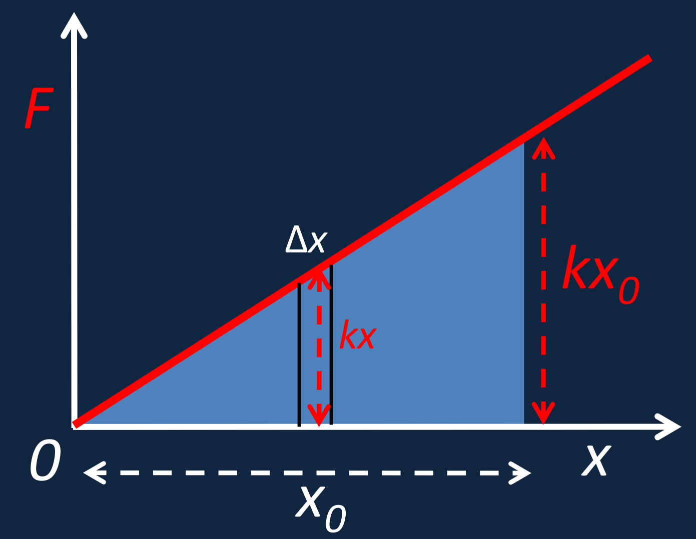


This work is stored by the spring as potential energy.

Total Work as *Area Under Curve*

- Plotting a graph of external force $F = kx$ as a function of *x*, the work to stretch the spring from x to $x + \Delta x$ is $kx\Delta x$, just the *incremental area under the curve*, so the total work is the total area

$$W = \int_0^{x_0} kx\,dx = \tfrac{1}{2}kx_0^2$$



Area under this "curve" =½ base x height= $\tfrac{1}{2}kx_0^2$
In fact, the total work done is the area under the force/distance curve for *any* curve: it's a sum of little areas $F\Delta x$ corresponding to work for Δx.

Problem from book

- **11.** (II) A 400-kg piano slides 4.0 m down a 30° incline and is kept from accelerating by a man who is pushing back on it *parallel to the incline*. Determine: (*a*) the force exerted by the man, (*b*) the work done by the man on the piano, (*c*) the work done by the force of gravity, and (*d*) the net work done on the piano. Ignore friction.

Problem from book

- **38.** (II) If it requires 5.0 J of work to stretch a particular spring by 2.0 cm from its equilibrium length, how much more work will be required to stretch it an additional 4.0 cm?

Problem from book

- **49.** (III) A 3.0-m-long steel chain is stretched out along the top level of a horizontal scaffold at a construction site, in such a way that 2.0 m of the chain remains on the top level and 1.0 m hangs vertically. At this point, the force on the hanging segment is sufficient to pull the entire chain over the edge. Once the chain is moving, the kinetic friction is so small that it can be neglected. How much work is performed on the chain by the force of gravity as the chain falls from the point where 2.0 m remains on the scaffold to the point where the entire chain has left the scaffold? (Assume that the chain has a linear weight density of 18N/m.)

Problem from book

- **48.** (III) A 3000-kg space vehicle, initially at rest, falls vertically from a height of 3200 km above the Earth's surface. Determine how much work is done by the force of gravity in bringing the vehicle to the Earth's surface.