# More Circular Motion

Physics 1425 Lecture 10

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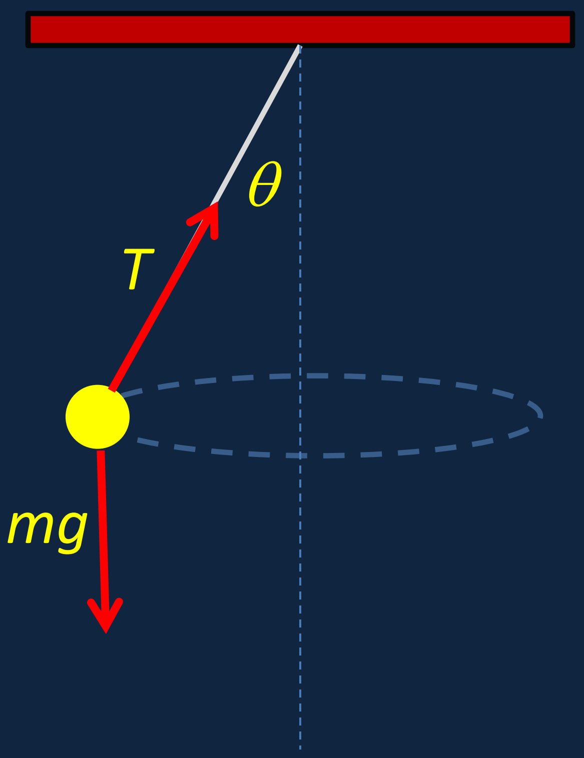
# The Conical Pendulum

- A mass moving in a horizontal circle, suspended by a string or rod from a fixed point above.
- If the tension in the string or rod is *T*, and the string is θ degrees from the vertical,

$$T \sin\theta = mv^2 / r,$$

$$T \cos\theta = mg,$$

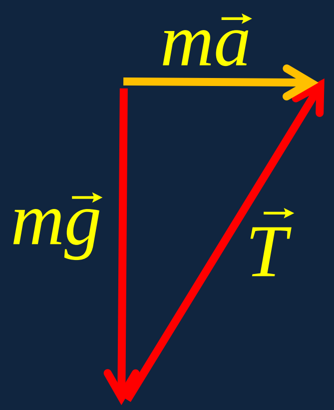
$$\tan\theta = v^2 / rg.$$

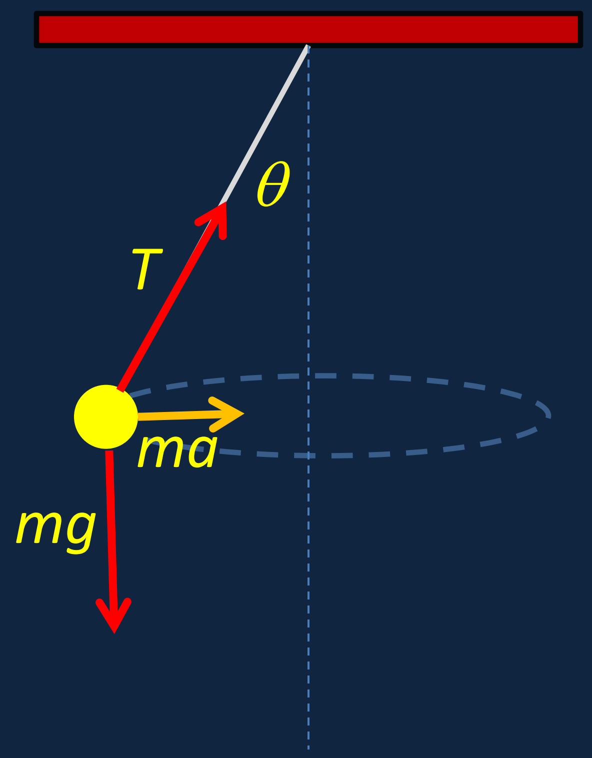


# $\vec{F} = m\vec{a}$ for the Conical Pendulum

- Notice how vector addition gives

$$\vec{F} = m\vec{g} + \vec{T} = m\vec{a}$$



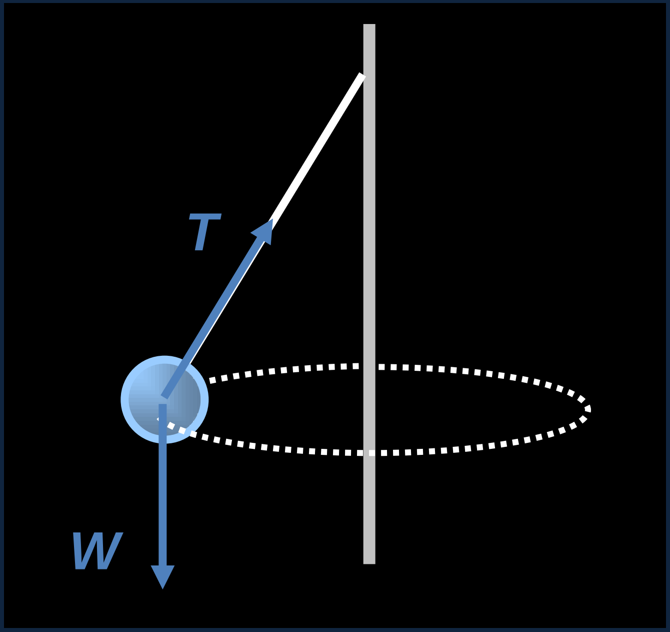
# Conical Pendulum as Control

- An early steam engine: as the conical pendulum rotates faster, driven by the engine, the masses rise and the levers cut back the steam supply.
- It can be preset to keep the engine within a given speed range.

# *ConcepTest 5.6* Tetherball

In the game of tetherball, the struck ball whirls around a pole. In what direction does the **net force** on the ball point?

1) toward the top of the pole
2) toward the ground
3) along the horizontal component of the tension force
4) along the vertical component of the tension force
5) tangential to the circle



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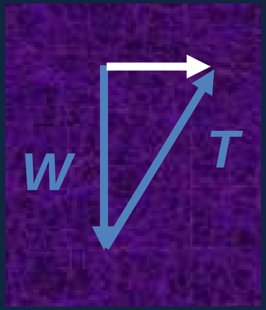
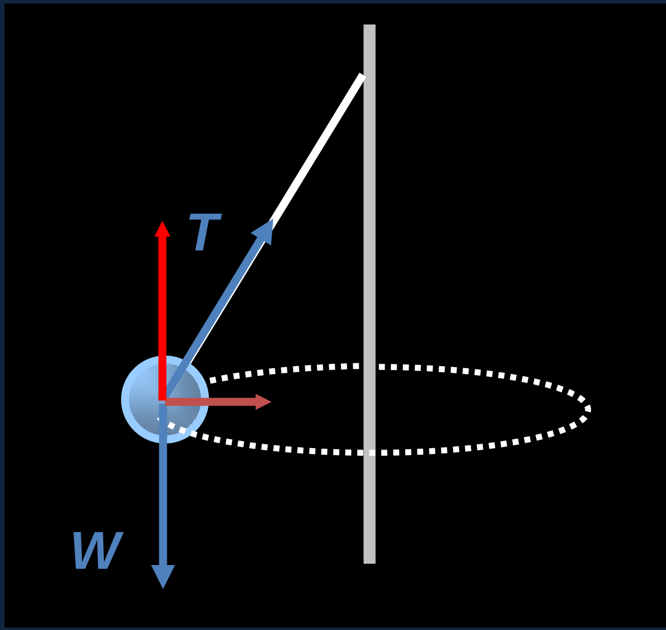
The **vertical component of the tension** balances the **weight**. The **horizontal component of tension** provides the **centripetal force** that points toward the center of the circle.



# *ConcepTest 5.9* Ball and String

Two equal-mass rocks tied to strings are whirled in horizontal circles. The radius of circle 2 is twice that of circle 1. If the period of motion is the same for both rocks, what is the tension in cord 2 compared to cord 1?

1) $T_2 = \frac{1}{4} T_1$
2) $T_2 = \frac{1}{2} T_1$
3) $T_2 = T_1$
4) $T_2 = 2T_1$
5) $T_2 = 4T_1$

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**The centripetal force in this case is given by the tension, so $T = mv^2/r$. For the same period, we find that $v_2 = 2v_1$ (and this term is squared). However, for the denominator, we see that $r_2 = 2r_1$ which gives us the relation $T_2 = 2T_1$.**

# *ConcepTest 5.7c* Around the Curve III

**You drive your dad's car too fast around a curve and the car starts to skid. What is the correct description of this situation?**

1) **car's engine is not strong enough to keep the car from being pushed out**
2) **friction between tires and road is not strong enough to keep car in a circle**
3) **car is too heavy to make the turn**
4) **a deer caused you to skid**
5) **none of the above**

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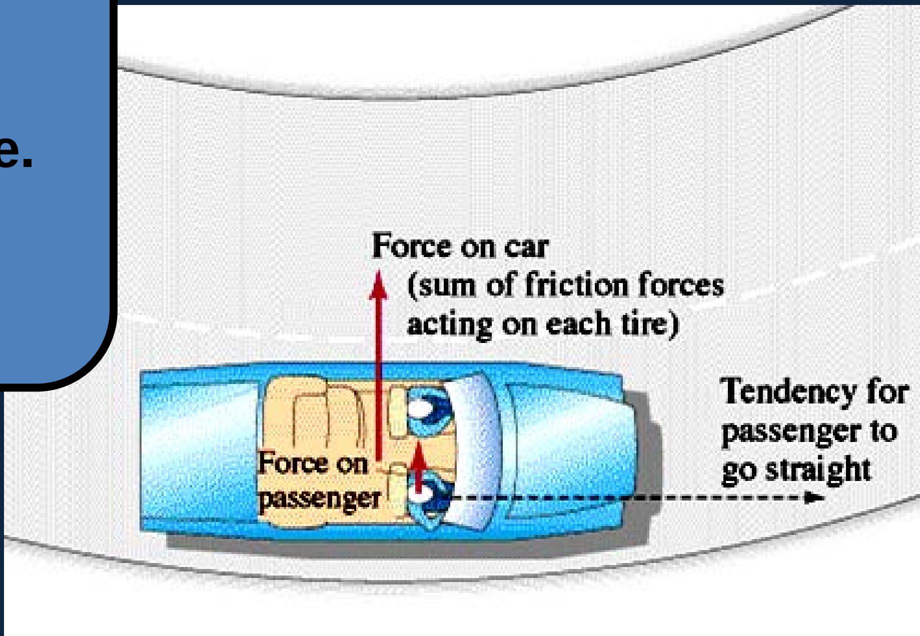
3) car is too heavy to make the turn
4) a deer caused you to skid
5) none of the above

The friction force between tires and road provides the centripetal force that keeps the car moving in a circle. If this force is too small, the car continues in a straight line!

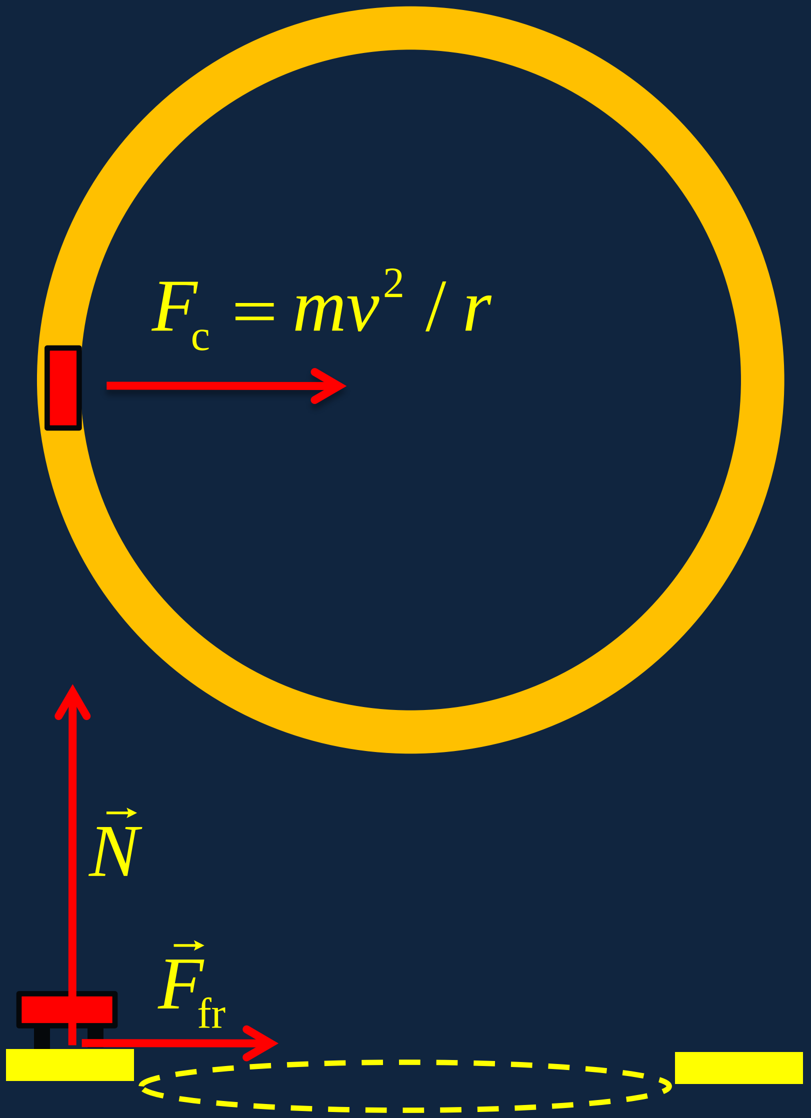



**Follow-up: What could be done to the road or car to prevent skidding?**

# Car on Flat Circular Road

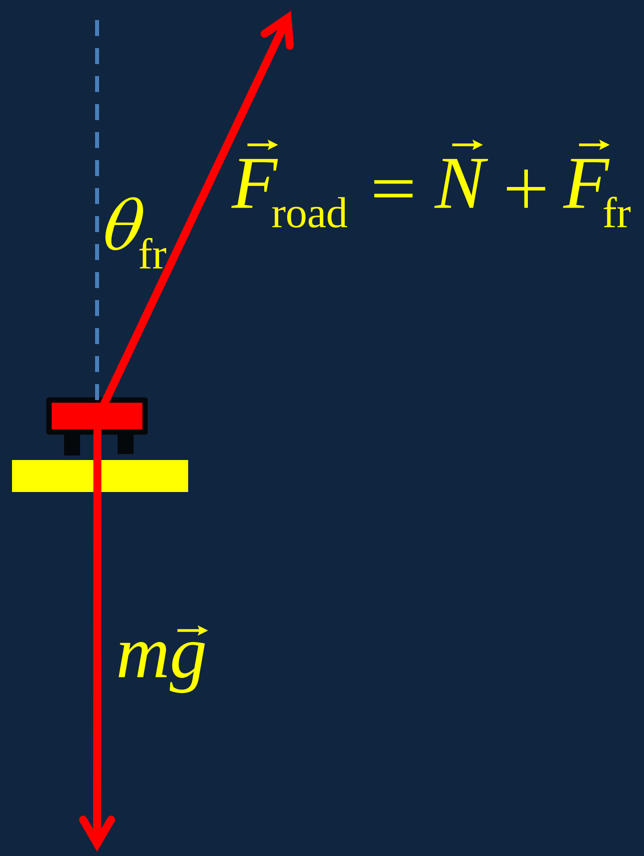
- For steady speed $v$ on a road of radius $r$, there must be a centripetal force $mv^2/r$.
- This is provided by friction between the tires and the road: at maximum nonskid speed

$$\boxed{F_{\text{fr}} = \mu_{\text{s}} N = \mu_{\text{s}} mg = mv^2 / r}$$



# Total Road Force on Car

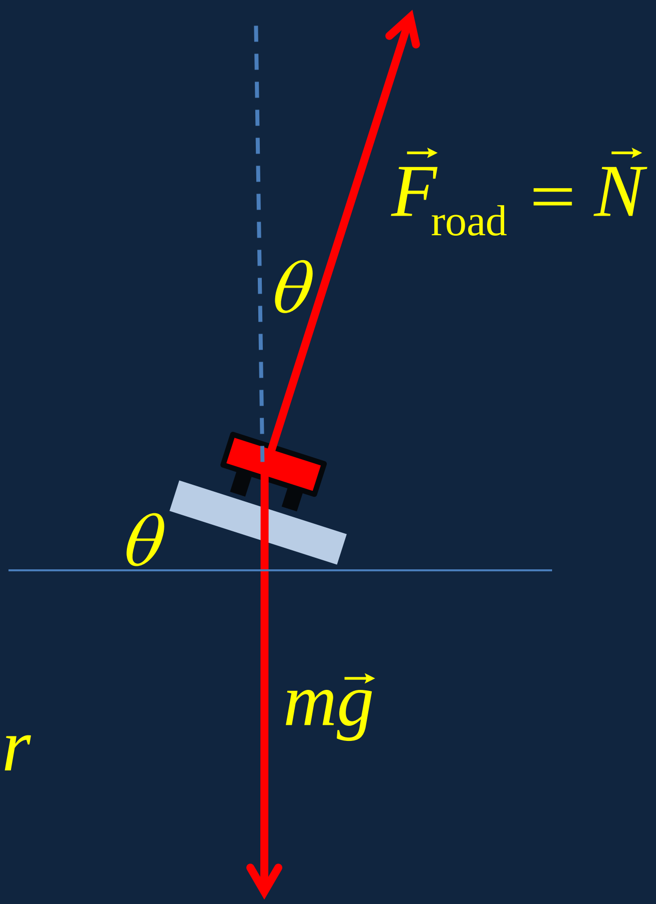
- The actual force $\vec{F}_{\text{road}}$ on the car from the road is the vector sum of the normal force and the frictional force.
- Notice the forces on the car have the same configuration as the conical pendulum!
- At maximum nonskid speed, $\vec{F}_{\text{road}}$ is at an angle $\theta_{\text{fr}}$,




$$\boxed{\tan\theta_{\text{fr}} = F_{\text{fr}} / N = \mu_s.}$$

# Banked Road: Sheet of Ice

- The **normal force** is always **perpendicular** to the road surface.
- Banking a curved road turns $\vec{N}$ inward to provide a centripetal force even at zero friction—but only for the *right speed*!




$$N\cos\theta = mg, \quad N\sin\theta = mv^2 / r$$

- So $\boxed{v^2 = rg\tan\theta}$ (the same as the conical pendulum)

# Maximum Speed on Banked Road

- At maximum speed, friction adds $\vec{F}_{\text{fr}}$ to $\vec{N}$ to give a total road force

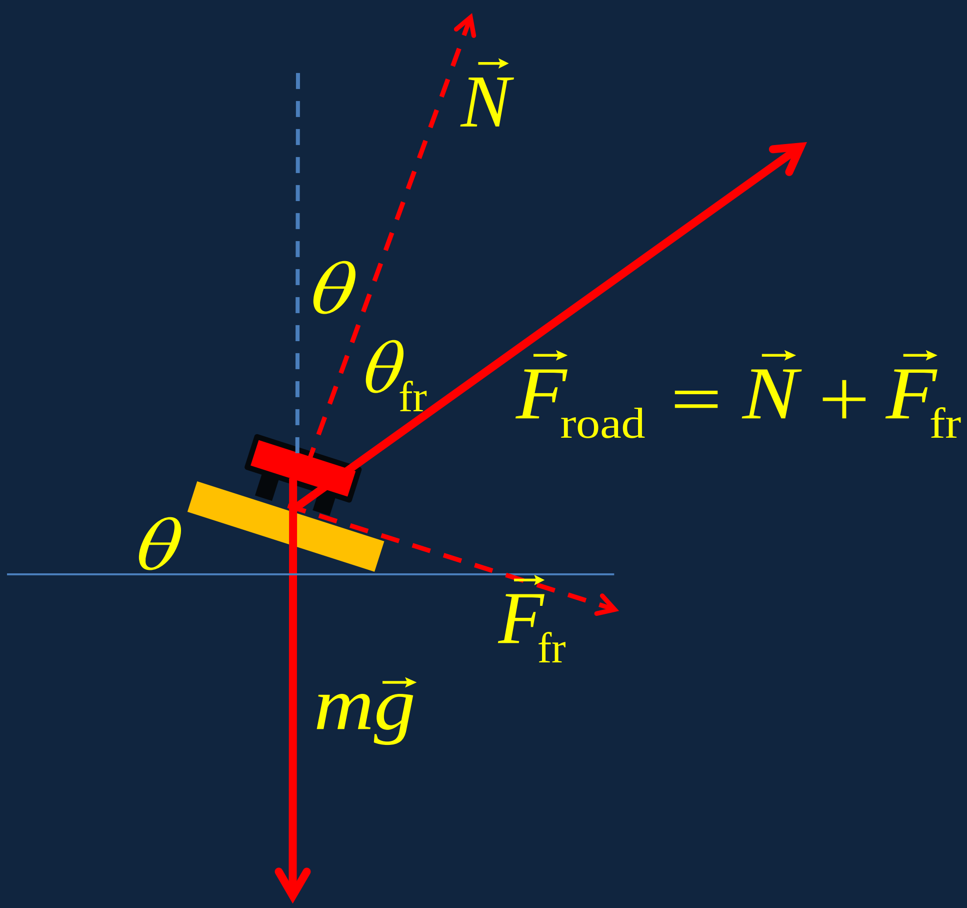
$$\vec{F}_{\text{road}} = \vec{N} + \vec{F}_{\text{fr}}$$

at an angle $\theta_{\text{fr}}$ to $\vec{N}$, where

$$\tan\theta_{\text{fr}} = F_{\text{fr}} / N = \mu_{\text{s}}.$$

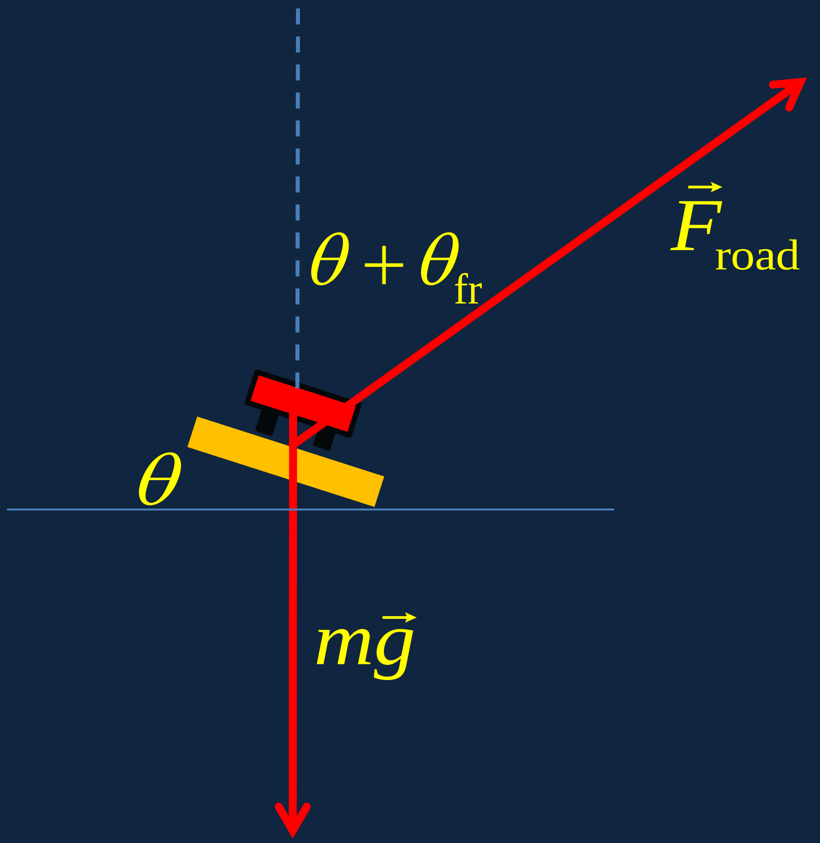
- The only forces acting on the car are $\vec{F}_{\text{road}}$ and $m\vec{g}$, so the conical pendulum equation is correct again:

$$v_{\max}^2 = rg\tan\left(\theta + \theta_{\text{fr}}\right)$$



# Maximum Speed on Banked Road

- Here are the two forces acting on the car, $\vec{F}_{\text{road}}$ and $m\vec{g}$ .
- Racing tires can have coefficient of friction $\mu_{\text{s}}$ close to 1, so from
  $\tan\theta_{\text{fr}} = F_{\text{fr}} / N = \mu_{\text{s}}$,
  $\theta_{\text{fr}}$ can be 45°.
- Now $v_{\max}^2 = rg\tan\left(\theta + \theta_{\text{fr}}\right)$, so for *banking angle* 45°, *and* $\mu_{\text{s}}$ = 1, $v_{\max}$ is infinite!




(Of course, as *v* becomes *very* large, so does the centripetal force **and therefore the normal force**—something will give!)

# Clicker Question

What is the direction of the acceleration of a pendulum at the furthest point of its swing?

A. Downwards.
B. In the direction it’s about to move.
C. No acceleration at this point.

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Consider how the velocity changes from the instant the pendulum is at rest until a very short time later. (Or, from a slightly *earlier* time, but watch the sign.)

# Clicker Question

What is the direction of acceleration of a pendulum at the midpoint of its swing?

A. Downwards
B. Upwards
C. Horizontal
D. No acceleration at this point.

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What is the direction of acceleration of a pendulum at the midpoint of its swing?

A. Downwards
B. Upwards
C. Horizontal
D. No acceleration at this point.

The pendulum is not picking up or losing speed at this point, so this is just circular motion.

# Clicker Question

What is the direction of acceleration of a pendulum halfway down from the furthest point towards the midpoint of its swing?

A. Downwards
B. Upwards
C. Along the path
D. At some angle to the path, pointing above the path.
E. At some angle to the path, pointing below the path.

# Nonuniform Circular Motion

- The swinging pendulum is an example of nonuniform circular motion, as is a car picking up speed on a curve.
- Remember acceleration is a vector: it has a component in the direction of motion (called the tangential component) equal to the rate of change of velocity in that direction—the car's acceleration along the road, $dv/dt$.
- It also has the usual $v^2/r$ centripetal component towards the center of the curve.

# Drag Forces

- There are two kinds of drag forces:
- Viscous drag, as in pushing something through molasses. This drag force is linear in $v$. It's relevant for tiny particles in air and water, and small bubbles in molasses, etc.
- Inertial drag: the effort involved in shoving air or water out of the way as you move through it. This is proportional to $v^2$, and this is the usual drag for cars, boats, etc.
- Terminal velocity: for a falling object, the speed at which the drag force equals $mg$, so no *net* force acts, the object falls at constant speed.