60. Use the coordinate system indicated in the diagram. We start with the conditions for momentum and kinetic energy conservation.

Before

After

*x*

*y*

 

 

 Note that from the kinetic energy relationship, since the right side of the equation is positive, we must have 

 Now we may eliminate by squaring the two momentum relationships and adding them.

 

 Combining the previous result with the conservation of energy result gives the following.

 

 

(*a*) Consider  If  its maximum value, then  As  decreases towards 0, eventually the first term in the expression for  will dominate, since it has  as a factor. That term will also be negative because  The expression for  will eventually become negative and approach  in a continuous fashion. Thus will for some value of  have the value of –1, indicating that there is some allowable value of  that causes  and so all scattering angles are possible. A plot of vs. is helpful in seeing this. Here is such a plot for  Note that it indicates that the speed of the incident particle will range from a minimum of about  for a complete backscatter (a one-dimensional collision) to , which essentially means a “miss” – no collision. We also see that the graph is monotonically decreasing, which means that there are no analytical extrema to consider in the analysis.

(*b*) Now consider  If  its maximum value, then again we will have  As  decreases towards 0, eventually the first term in the expression for  will dominate, since it has  as a factor. But both terms in the expression are positive, since  So the expression for  will eventually approach  in a continuous fashion, and will never be negative. Thus there will not be any scattering angles bigger than  in any case. But is there a maximum angle, corresponding to a minimum value of  We look for such a point by calculating the derivative 

 

 Using this critical value gives the following value for  which we label as 

 

This gives the largest possible scattering angle for the given mass ratio. Again, a plot is instructive. Here is such a plot for  We find the maximum scattering angle according to the equation above.

 

The equation and the graph agree. The spreadsheet used for this problem can be found on the Media Manager, with filename “PSE4\_ISM\_CH09.XLS,” on tab “Problem 9.60b.”