Optical Instruments

Physics 2415 Lecture 34

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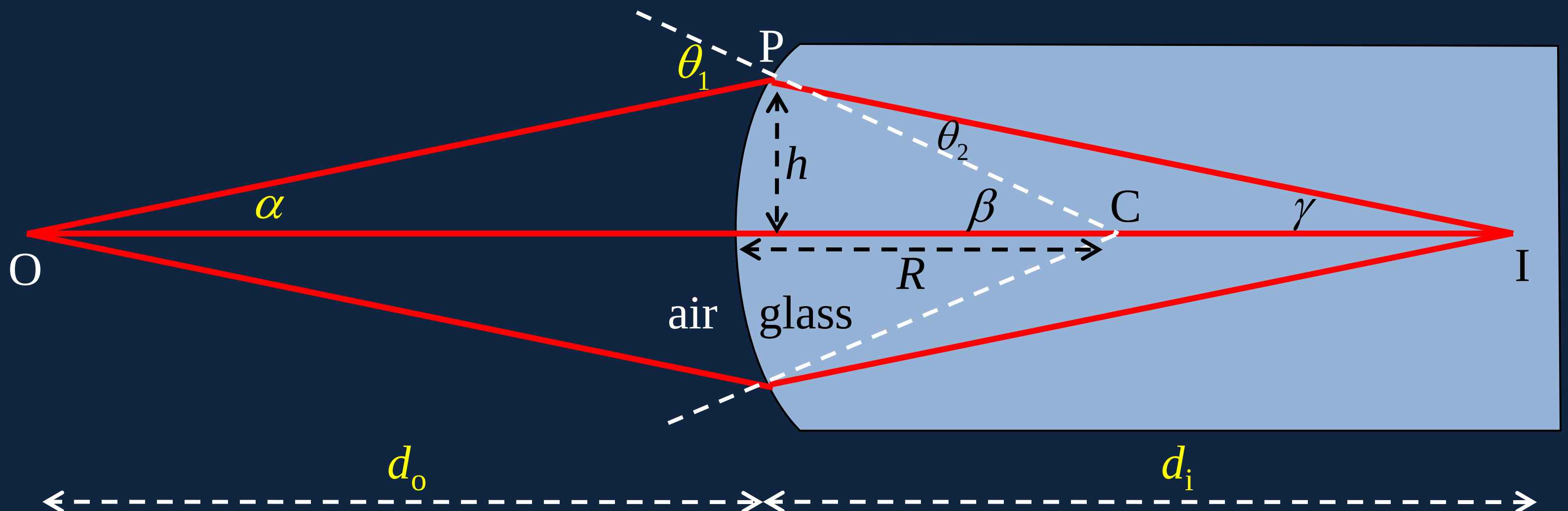
Today’s Topics

- The lensmaker’s formula
- Magnifying power
- Lens combinations: ray tracing, telescopes.

Refraction at a Spherical Surface

- Rays close to the axis ("paraxial") will focus to an image inside the glass:



- From $\theta_1 = n\theta_2 = \alpha + \beta = \alpha + \theta_2 + \gamma, \;\; h = d_o\alpha = R\beta = d_i\gamma$

  we can show that $\dfrac{1}{d_o} + \dfrac{n}{d_i} = \dfrac{n-1}{R}$

The Lensmaker's Formula

(optional derivation, if you're curious)

- The formula $\frac{1}{d_o}+\frac{n}{d_i}=\frac{n-1}{R}$ also works <u>in reverse</u>. A ray coming from an object in the glass will satisfy $\frac{n}{d_o}+\frac{1}{d_i}=\frac{n-1}{R}$
- For a convex lens with surfaces of radii R_1, R_2, the rays on going through R_1 will converge (inside the glass) towards a point d_1 such that $\boxed{\frac{1}{d_o}+\frac{n}{d_1}=\frac{n-1}{R_1}}$.
- But those rays don't get there—they first meet surface R_2, which focuses them in air to a point d_i, say, these rays being from a <u>virtual object</u> at d_1, so the object distance is $-d_1$, the final image is at d_i: $\boxed{-\frac{n}{d_1}+\frac{1}{d_i}=\frac{n-1}{R_2}}$
- Adding the boxed formulas gives:

$$\frac{1}{d_o}+\frac{1}{d_i}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=\frac{1}{f}$$

The Lensmaker's Formula

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

- This formula also works for plano convex lenses (one side flat, meaning R infinite) or if one or both sides are concave—but for concave sides, R must be taken negative.

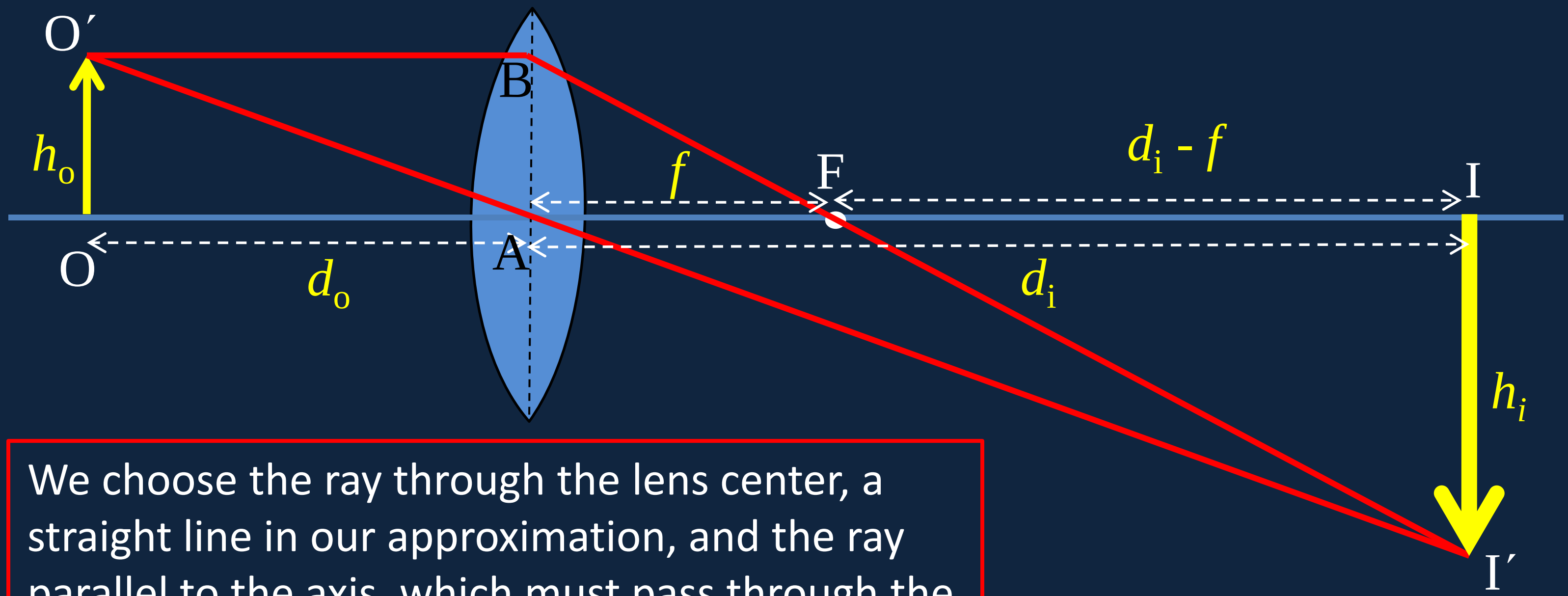
- Note: sometimes this formula is written with a minus sign—in those books, the rule is that R is taken positive if its center of curvature over is to the right. It's a matter of taste.

Image Location by Ray Tracing

- The rules we use for thin lenses:

1. We take the ray through the center of the lens to be undeflected and unshifted.
2. For a convex lens, rays passing through a focus on one side are parallel to the axis on the other side.
3. For a concave lens, rays coming in parallel on one side are deflected so they apparently come from the focal point on that same side.

Ray Tracing for a Thin Convex Lens



We choose the ray through the lens center, a straight line in our approximation, and the ray parallel to the axis, which must pass through the focus when deflected. They meet at the image.

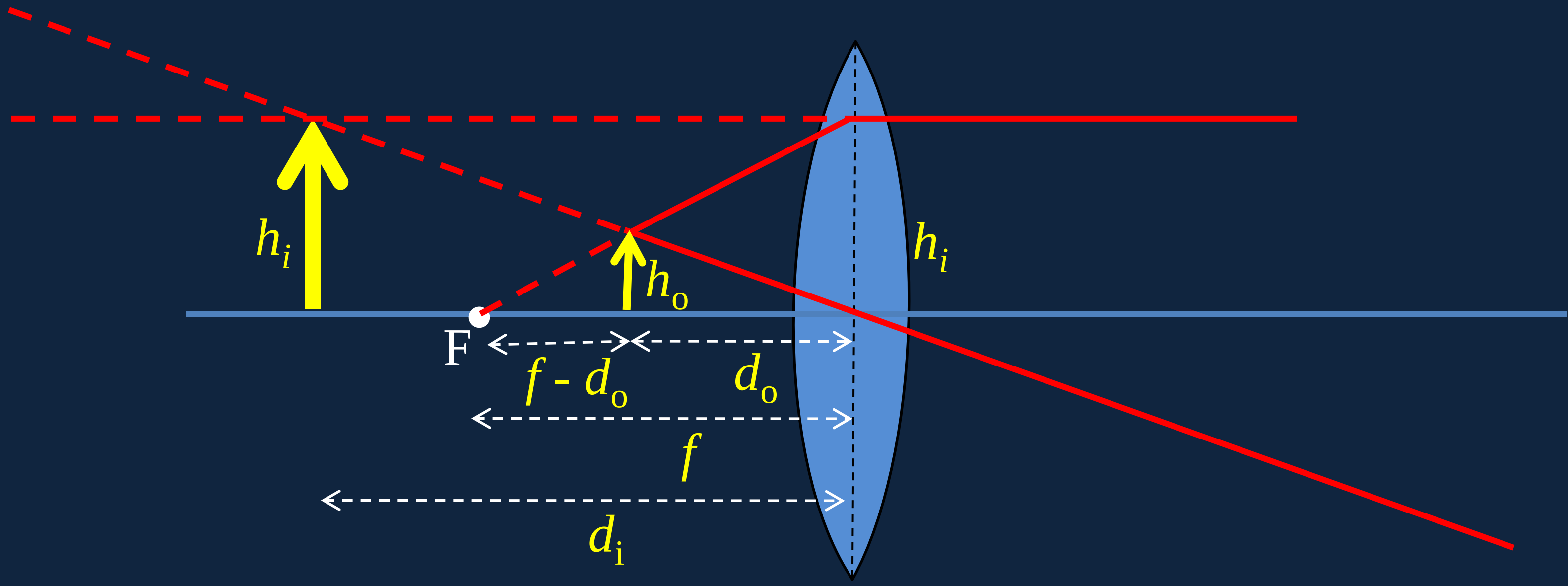
From the straight line through the center, $h_o / d_o = h_i / d_i, \quad h_o / h_i = d_o / d_i$

from the line BFI´ (and similar triangles!), $h_o / h_i = \mathrm{BA} / h_i = f / (d_i - f)$

This gives immediately: $$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

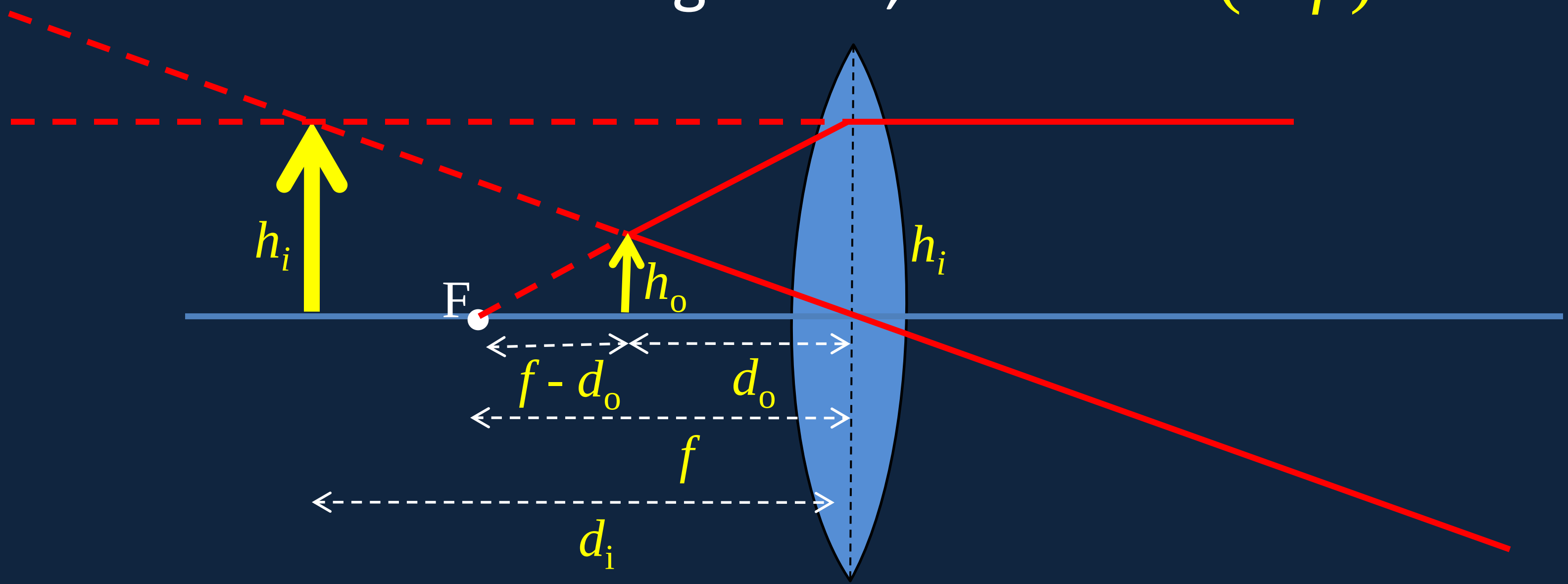
Convex Lens as Magnifying Glass

- The object is closer to the lens than the focal point F. To find the virtual image, we take one ray through the center (giving $h_i / h_o = d_i / d_o$) and one through the focus near the object ($h_i / h_o = f / (f - d_o)$), again $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ but now the (virtual) image distance is taken <u>negative</u>.



Definition of Magnifying Power

- M is defined as the ratio of the angular size of the image to the angular size of the object observed with the naked eye at the eye's near point N, which is h_o/N.
- If the image is at infinity ("relaxed eye") the object is at f, the magnification is $(h_o/f)/(h_o/N) = N/f$.
- Maximum M is for image at N, then $M = (N/f) + 1$.



Simple and Compound Microscopes

- The simple microscope is a single convex lens, of very short focal length. The optics are just those of the magnifying glass discussed above.
- The simplest compound microscope has two convex lenses: the first (objective) forms a real (inverted) image, the second (eyepiece) acts as a magnifying glass to examine that image.
- The total magnification is a product of the two: the eyepiece is N/f_e, N = 25 cm (relaxed eye) the objective magnification depends on the distance ℓ between the two lenses, since the image it forms is in the focal plane of the eyepiece.

Diverging (Concave) Lens

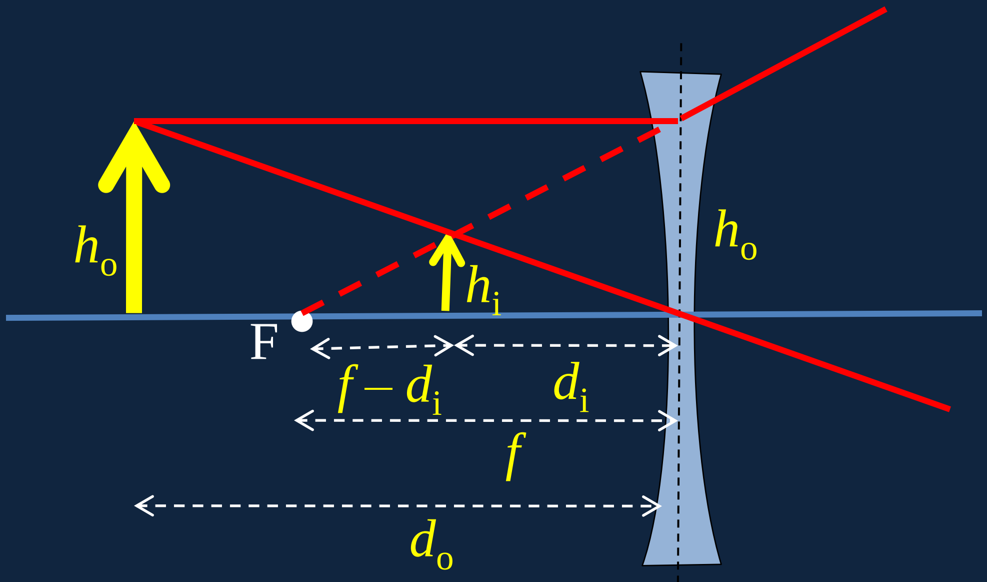
- The same similar triangles arguments here give

$$\frac{h_o}{h_i} = \frac{d_0}{d_i} = \frac{f}{f - d_i}$$

from which

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

provided we now take both d_i and f as <u>negative!</u>



Formula Rules Updated…

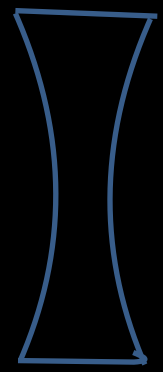
- The formula

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}$$

  is valid for any thin lens.
- For a converging lens, f is positive, for a diverging lens f is negative.
- The object distance d_{o} is positive—unless, *in a multi-lens system*, the object is on the "wrong" side of the lens! (We'll do an example.)
- The image distance d_{i} is positive for a real image, negative for a virtual image.

Empty Lens

A "concave lens" is actually made of very thin glass, is hollow and filled with air. How will this lens behave at close quarters under water?



1) It will magnify

2) Things will look smaller

3) Things will look the same size

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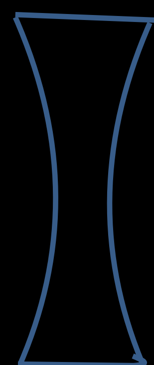

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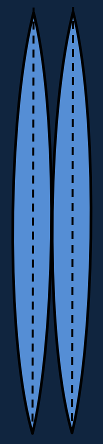
Clicker Question

- I have two identical thin convex lenses of focal length f. If I put them together ()(), what is the focal length of the combination?

A. $2f$

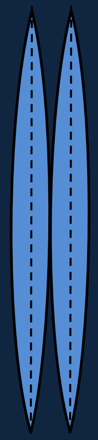
B. f

C. $f/2$



Clicker Answer

- $f/2$: the first lens refracts the rays towards a focus at f, they immediately encounter the second lens, which refracts them more, to a closer focus.



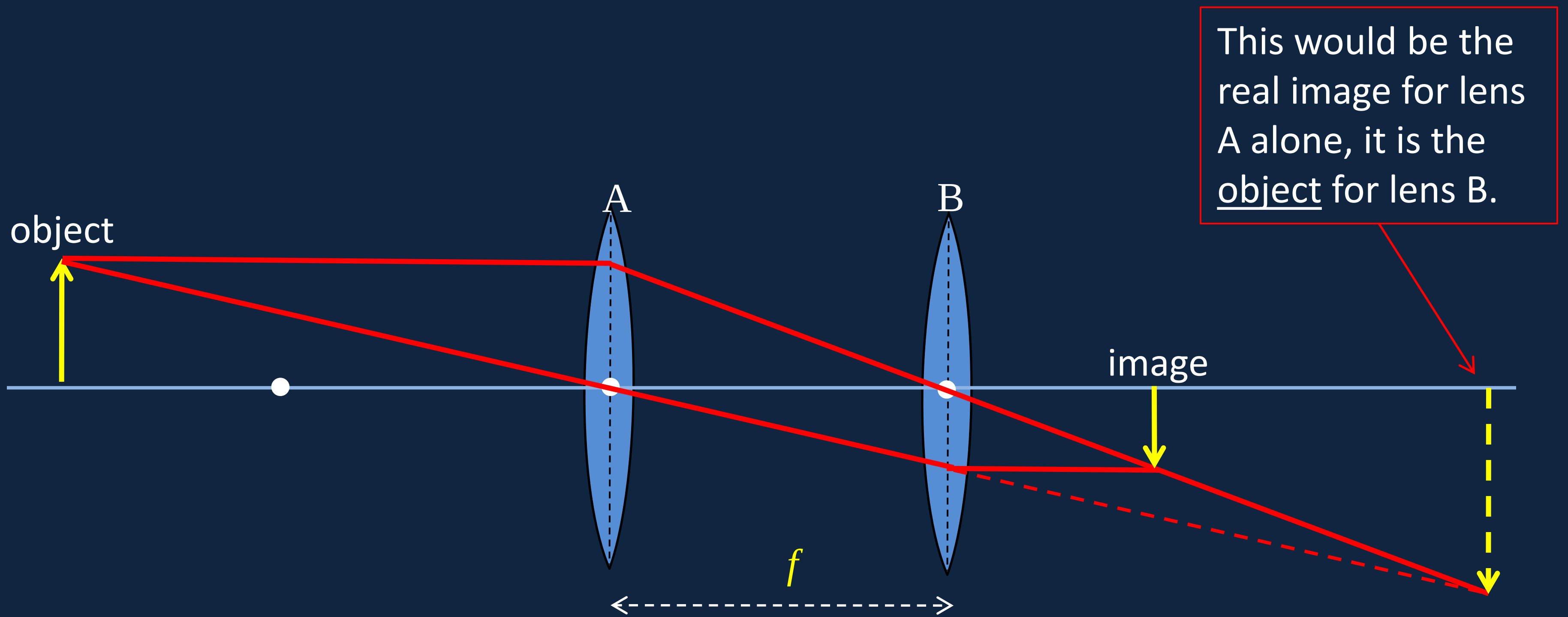
- Important! The image from the first lens is the object for the second lens.

- Combined focal length from formula: for the second lens, $d_o = -f$, the object is *behind* the lens!

- From $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$, we have $-\frac{1}{f} + \frac{1}{d_i} = \frac{1}{f}, \quad d_i = \frac{f}{2}.$

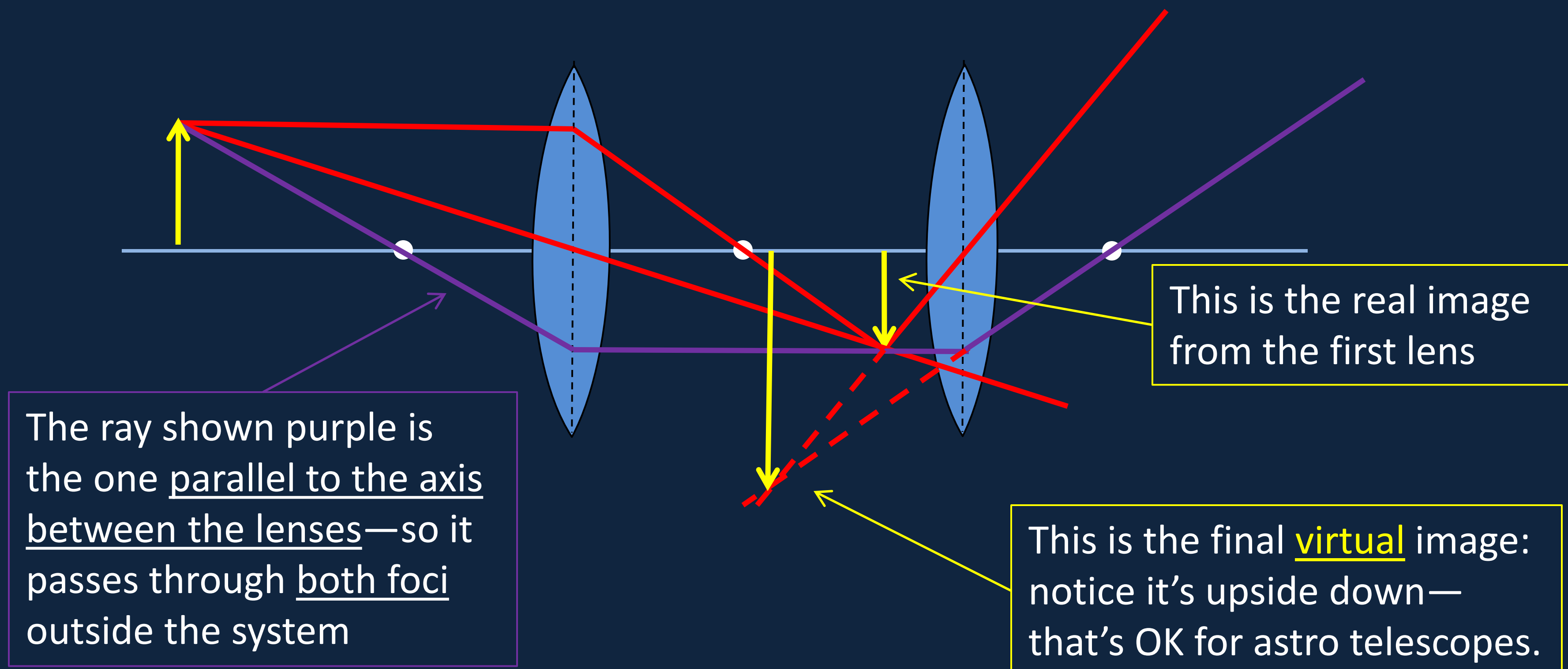
Two Convex Lenses Separated

- Easy example: two lenses, same focal length f, separated by f, so rays through the center of one lens are parallel to the axis after (or before) passing through the other lens:



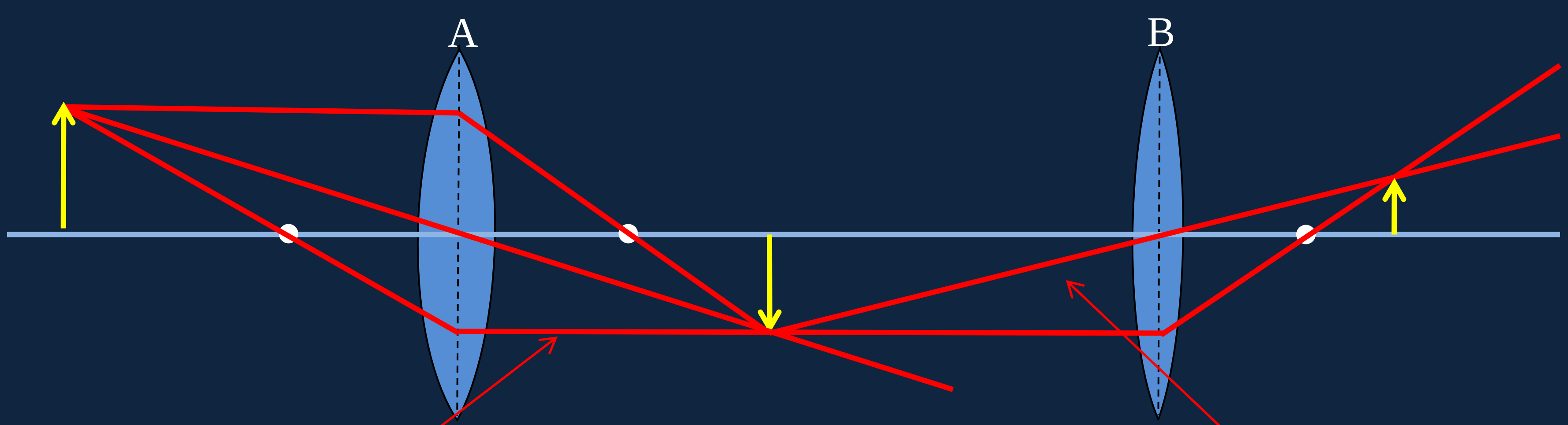
Further Separated…

- If the first lens forms an image between the lenses, but less than the focal distance to the second lens, the combination produces a virtual image (this is the basic ray pattern for simple telescopes and microscopes):



Even More Separated…

- If the separation is sufficient that the image from the first lens A is outside the focal length of lens B, there is a final real upright image beyond the second lens:

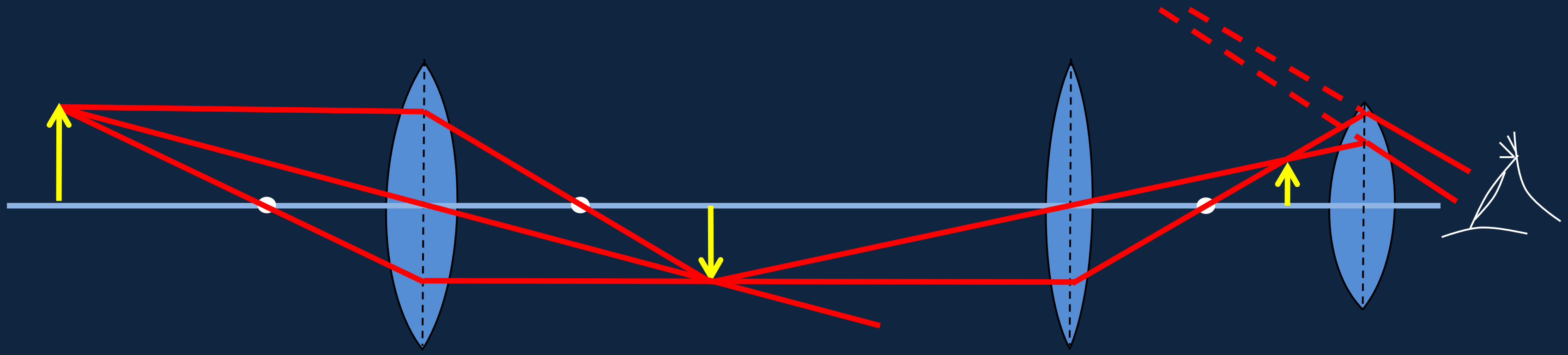


Notice the usefulness of the ray parallel to the axis between the lenses—it goes through the foci (white circles) outside.

We first locate the image from lens A, then draw in the ray from it through the center of lens B

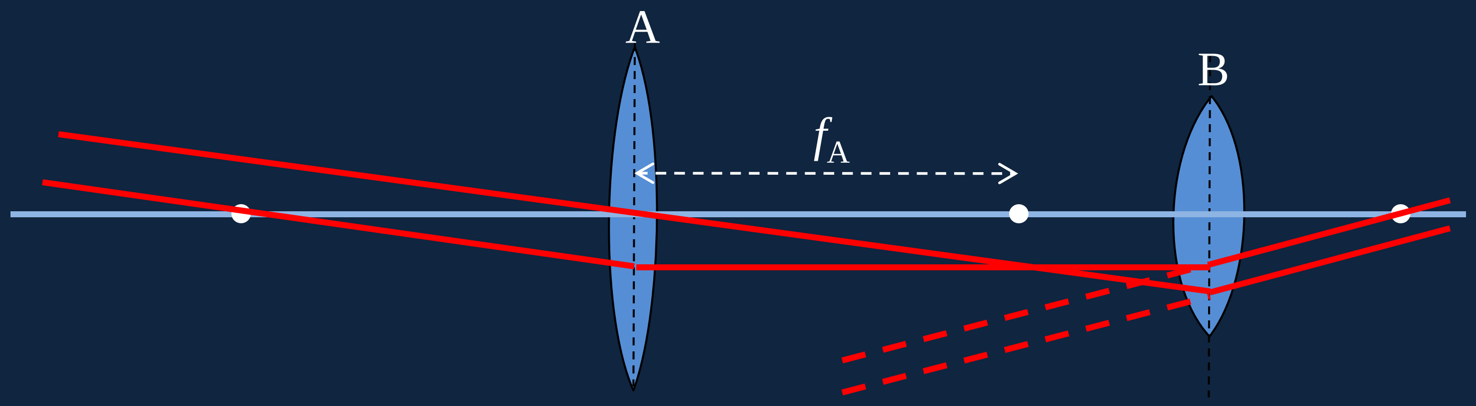
The Spyglass

- The real image from the two convex lenses can be viewed through a third, powerful, lens to make a telescope with upright image, better for terrestrial viewing (as opposed to astronomical uses).



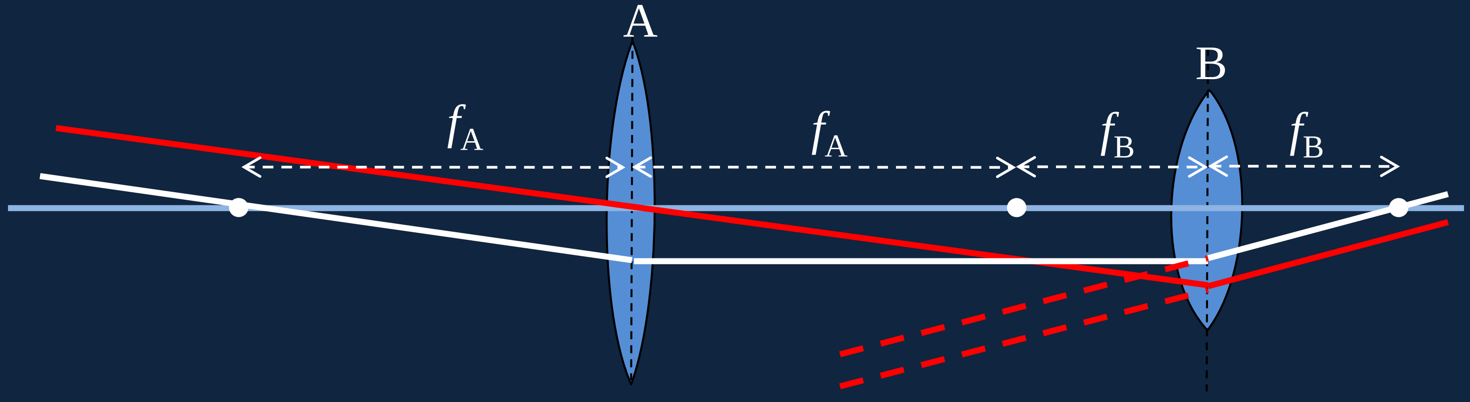
Astronomical Telescope: <u>Angular</u> Magnification

- Any object in astronomy can be taken to be at infinite distance: the relevant image size parameter is the angular size of the image.
- Example: imagine pointing a telescope at Jupiter, so Jupiter's south pole is on the axis of the telescope.
- Rays coming from Jupiter's north pole can be taken to be parallel and at a small angle to the axis on entering the telescope, so they form an image in the focal plane…



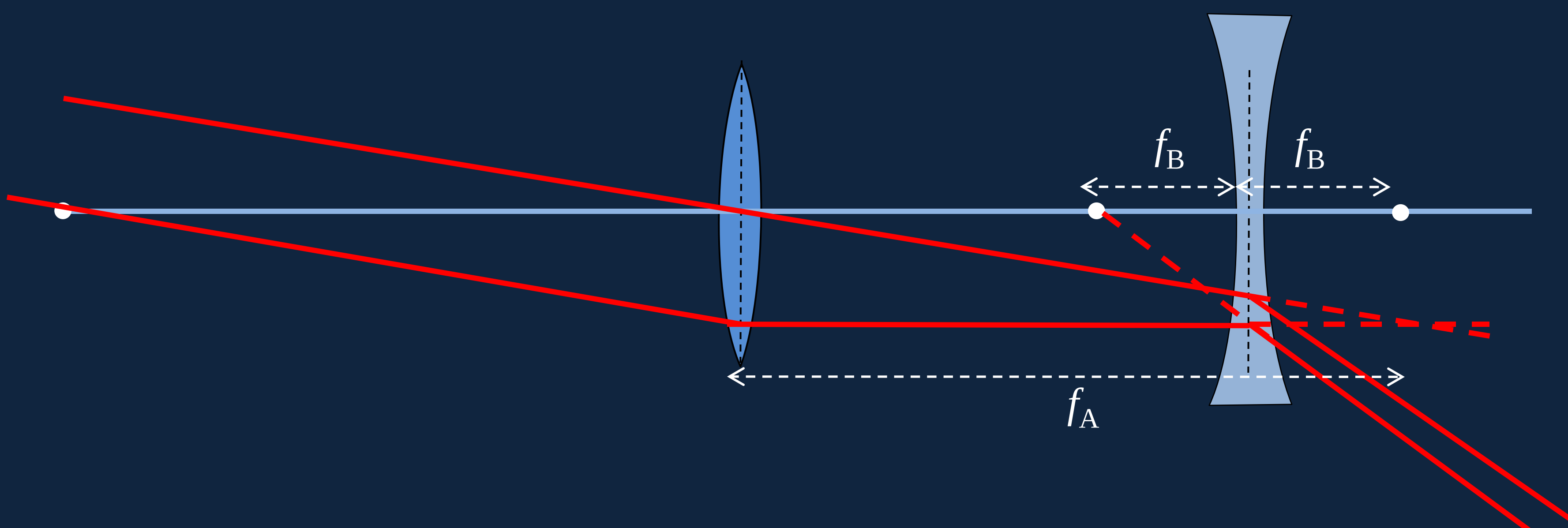
Astronomical Telescope: Angular Magnification

- An “eyepiece” lens of shorter focal length is added, with the image from lens A in the focal plane of lens B as well, so viewing through B gives an image at infinity.
- Tracking the special ray that is parallel to the axis between the lenses (shown in white) the ratio of the angular size image/object, the magnification, is just the ratio of the focal lengths f_A/f_B.

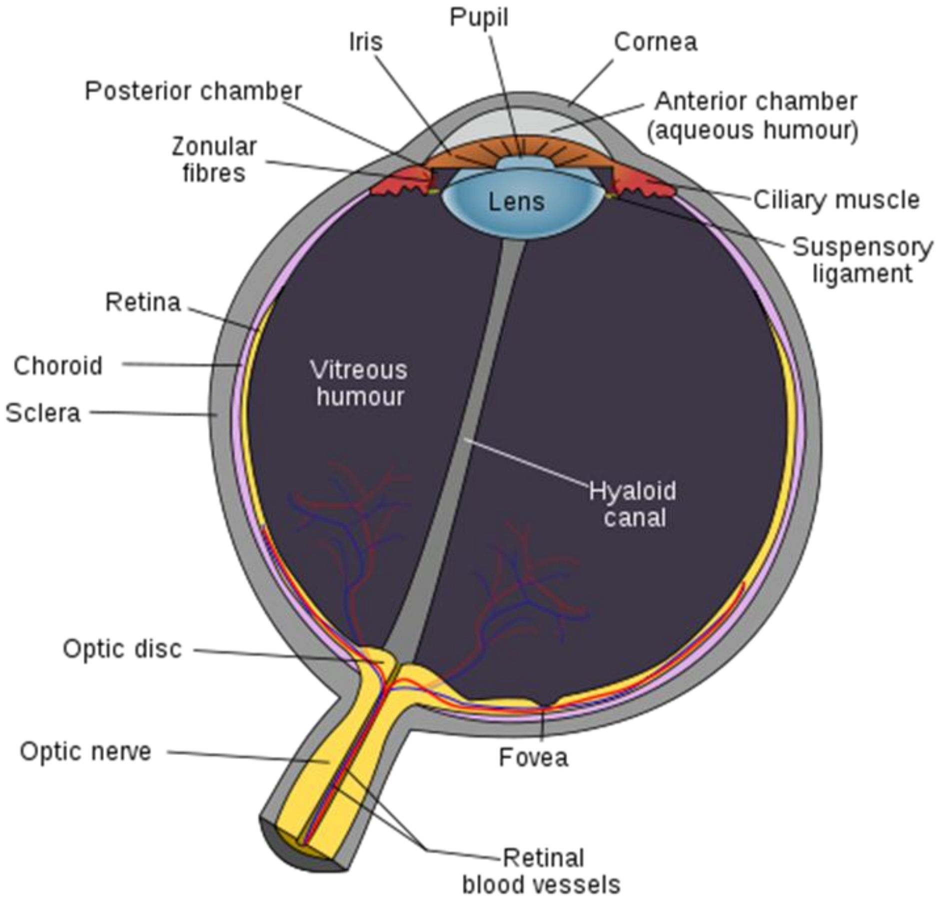


Galilean Telescope

- The rays from the object lens are intercepted by a concave lens before they form an image. The concave lens is positioned so that the image would have been at its focus—so it forms a virtual image at infinity (from the lens formula).
- The angular magnification is again the ratio of focal lengths.



The Eye



Most of the focusing takes place at the cornea, filled with watery stuff.
The lens shape is adjusted by muscles to make finer adjustments to the focusing.