Electromagnetic Waves and Polarization

Physics 2415 Lecture 30

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Today’s Topics

• Measuring the speed of light
• Wave energy and power: the Poynting vector
• Light momentum and radiation pressure
• Radio transmission
• Polarization
Roemer, in 1676, noticed that eclipses of Jupiter’s satellite Io gradually fell behind a regular schedule for about half the year, then just as gradually caught back up. He figured it was because the Earth was going away from Jupiter for a little over half a year, then approaching again: it’s a Doppler effect! He got a reasonable estimate of the speed of light.
First Measurement on Earth

Light goes out between teeth of rotating wheel, reflects off distant mirror, by the time it gets back, a tooth may be blocking its path, depending on wheel speed: at certain speeds, the observer sees nothing. Knowing the wheel rotation rate, the speed of light can be figured out. (1849)
Best Early Measurements of the Speed of Light (1879)...

- were by a physics instructor, Albert Michelson, at the Naval College in Annapolis—his rich father-in-law (who had a castle in Scarsdale) bankrolled the experiment. (It was within about 50 km/sec.)
- The velocity of light $c$ is precisely:
  $$c = 2.99792458 \times 10^8 \text{ m/sec.}$$
- This is *exact*, because it’s the definition of the meter. (*The second* is defined as a precise number of oscillations of a particular atom.)
The speed of light is $2.99792458 \times 10^8$ m/sec relative to:

1) You
2) Center of the Earth
3) Center of the Sun
4) Center of the Universe
5) All of the above
Speed of Light

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That’s the Theory of Special Relativity!

...but it’s not in this course, so you’ll all get three points.
Wave Energy

• Recall we found the energy/meter in a vibrating string, then multiplying that by the wave speed gave power delivered by the wave.
• The same analysis works for electromagnetic waves: for a harmonic wave \( E = E_0 \sin (kx - \omega t) \) and (from the previous lecture) \( E = \nu B = cB \).
• The energy density \( u \) (\( E, B \) mean \( \text{rms} \) values!) is

\[
u = \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2} B^2 / \mu_0 = \varepsilon_0 E^2 = c\varepsilon_0 EB
\]

(using \( B = E / c, \ c = 1 / \sqrt{\mu_0 \varepsilon_0} \) to see magnetic energy = electric).
Wave Power

- The energy density in the harmonic wave is equal to $u = c\varepsilon_0EB$ joules/m$^3$. ($c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$.)
- The power intensity—the energy delivered across one square meter perpendicular to the direction of the wave—equals the energy in a volume with a base of 1 sq m, a length of $c = 3\times10^8$ meters.
- Intensity $S = cu = c\varepsilon_0E^2 = c^2\varepsilon_0EB = EB / \mu_0$
- These terms all represent power/m$^2$ in watts.
Poynting Vector

• The energy density in the harmonic wave is equal to \( u = c \varepsilon_0 EB \) joules/m\(^3\).
• Intensity—power delivery/m\(^2\) —is \( S = \frac{EB}{\mu_0} \).
• Obviously, this power is delivered in a particular direction (that of the wave) and can be represented as a vector:

\[
\vec{S} = \left( \frac{1}{\mu_0} \right) \left( \vec{E} \times \vec{B} \right)
\]

This is called the Poynting vector.
**Poynting Vector for Static Fields**

• The Poynting vector gives energy flow even for *static* fields.

• If an electric field \( E \) is driving a steady current \( I \) along a wire, there will be a magnetic field \( B = \mu_0 I / 2\pi r \), and so a Poynting flow \( \mathbf{S} = (\mathbf{E} \times \mathbf{B}) / \mu_0 \) inwards as shown here.

• Flow across a cylindrical surface of radius \( r \), length 1 meter, will be \( A \mathbf{E} B / \mu_0 \), \( A = \text{area} = 2\pi r \), so total energy flow rate = \( EI \). This is just the power dissipated as heat!
Light has Momentum

• Maxwell proved from his equations that a flash of light with total energy $E$ carries momentum $E/c$.

• The proof is quite difficult, but one way to see the result is to use Einstein’s equation $E = mc^2$. The energy $E$ in a flash of a beam of light means it has a (very tiny!) mass $E/c^2$, moving of course at $c$, so momentum $p = mv = mc = E/c$.

• All you need to know is the result.
Radiation Pressure

• Since light carries momentum, anything absorbing or reflecting light feels a pressure, a force equal to the rate of change of momentum, from Newton’s laws.

• How can the perpendicular $E$ and $B$ fields push something forwards?

• The electric field causes charged particles (electrons) to oscillate perpendicular to the wave direction, then the force $q\vec{v} \times \vec{B}$ from the magnetic field pushes the charge forwards.
Radiation Pressure

• The equation $p = \frac{E}{c}$ means that it takes a lot of energy to get much momentum.

• The only known successful antisatellite laser, called MIRACL, delivered one megawatt continuous power to an area of 200cm$^2$.

• If all that energy hits a satellite, very approximately what force does it exert?

   A. 100N  B. 10N  C. 1N  D. 0.1N  E. 0.01N
Radiation Pressure

If one megawatt hits a satellite, very approximately what force does it exert?

E. 0.01N approximately—$p = \frac{E}{c}$ means 1 Mw, or $10^6$J/sec, is momentum $10^6/3x10^8 \sim 3x10^{-3}$ delivered per second, and force is rate of change of momentum, so this is $3x10^{-3}$N. If the radiation is reflected, the force is double this.

The point is to fry the satellite, not to push it!
Radio Transmission

• The basic radio transmitter is an oscillating dipole: at some instant, a dipole is created, its field propagates outwards, but it rapidly dies to be replaced by a dipole in the opposite direction—the outgoing electric field must switch direction, it does this by looping around as seen here. The magnetic field lines from current up and down the dipole antenna are circular.

Some animations
Radio Reception

• Although the radio wave looks complicated near the transmitter, far away (meaning more than a few wavelengths) it has the familiar form shown above, the direction of propagation being directly away from the source.

• For the wave shown above, generated by a vertical transmitting antenna, reception would be best with a vertical receiving antenna. The oscillating vertical electric field would set up oscillating currents in a vertical wire.
Radio Reception

• A radio receiving antenna has simultaneously many small oscillating currents, from all the transmitters within range.

• The antenna is linked to an $LC$ circuit with tuneable oscillation frequency—this circuit is driven by the antenna current with the right frequency, the signal is then amplified.

• Frequencies vary from kHz to GHz, and $\lambda f = c$ gives the corresponding wavelengths.
Polarization

- The electromagnetic wave shown above is said to be vertically polarized—meaning the direction of the electric field vector is vertical.

- A wave like this with the electric field vector at an angle $\theta$ to the vertical can be represented as a sum of a vertically polarized wave of amplitude $E \cos \theta$ and a horizontally polarized wave of amplitude $E \sin \theta$. 
Polarizing Light

• Ordinary light is a random mixture of polarizations. Certain materials, like polaroid, only allow light polarized, say, vertically, to pass.

• These materials have long horizontal electrically conducting molecules, so the horizontal component of the electric field is absorbed driving currents in these molecules.
How Much Intensity Gets Through Polaroid?

• For a wave polarized at $\theta$ to the vertical, only the component $E \cos \theta$ gets through. This means the intensity is down by a factor $\cos^2 \theta$.

• For incoming light with random polarizations, the reduction in intensity will be $\cos^2 \theta = \frac{1}{2}$. 
Polarization by Reflection

• We’ll discuss this in more detail later, but just mention here that reflected light is partially polarized, that from a flat horizontal surface partially horizontally polarized—polaroid sunglasses cut this out.

• Light reflected from a surface between two materials with different refractive indices $n_1$, $n_2$ is fully polarized if reflected at Brewster’s angle, given by $\tan \theta_p = n_2 / n_1$. 