Sound II

Physics 2415 Lecture 28

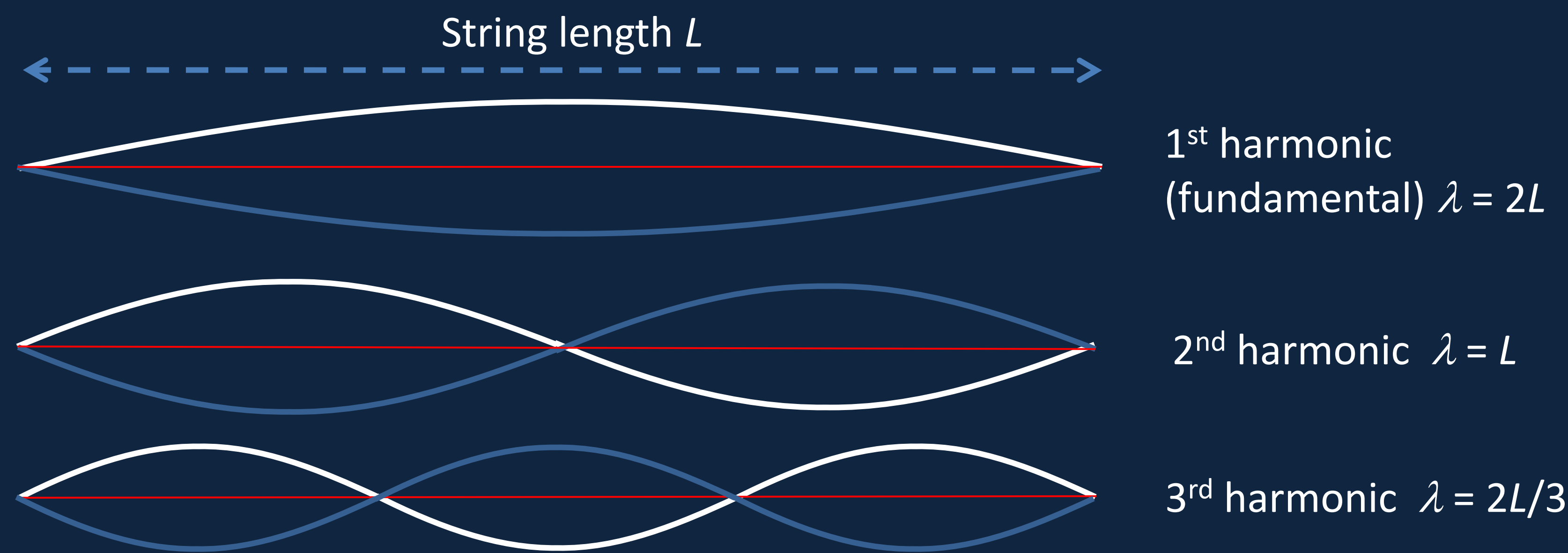
Michael Fowler, UVa

Today's Topics

- Waves in two and three dimensions
- Interference
- Doppler effect

Harmonic String Vibrations

- Strings in musical instruments have fixed ends, so pure harmonic (single frequency) vibrations are sine waves with a whole number of half-wavelengths between the ends. Remember frequency and wavelength are related by $\boxed{\lambda f = v}$!

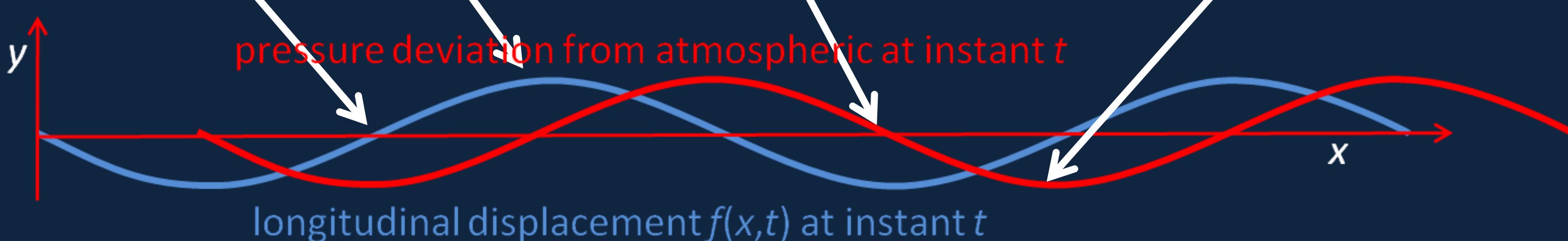


Longitudinal Harmonic Waves in Pipes

- What are possible wavelengths of standing harmonic waves in an organ pipe?
- Unlike standard string instruments, organ pipes can have two different types of end: closed and open.
- Obviously, longitudinal vibrations have no room to move at a closed end: this is the same as a fixed end for a transversely vibrating string.
- But what does the wave do at an open end?

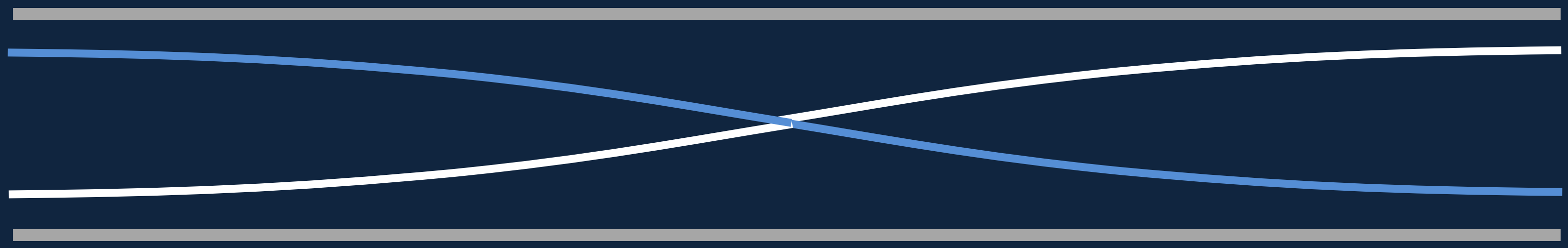
Boundary Condition at Pipe Open End

- At an open end of a pipe, the air is in contact with the atmosphere—so it's at atmospheric pressure.
- The boundary condition at the open end is that the pressure is constant, that is, $\Delta P = 0$.
- This means the amplitude of longitudinal oscillation is at a <u>maximum</u> at the open end!
- Node   Antinode   Pressure node   Pressure antinode



Clicker Question

- For an organ pipe with both ends open, the lowest note (fundamental) has $\lambda = 2L$.



- What is the wavelength of the next-lowest note (the second harmonic)?
- A. $\lambda = 3L$ B. $\lambda = (3/2)L$ C. $\lambda = L$ D. $\lambda = (2/3)L$

Clicker Answer

- Both ends open: second harmonic has $\lambda = L$.

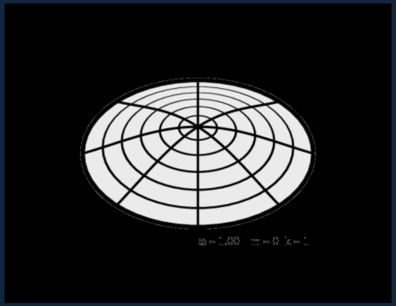
Waves in Two and Three Dimensions

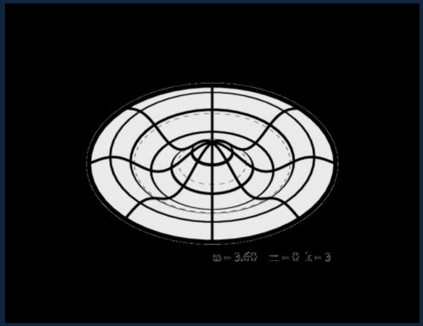
- Recall that the wave equation for waves on a string was given by matching the mass x accn for a tiny piece of string with the tension force from the two ends not being quite parallel.
- A similar argument applied to a tiny square part of a drumhead gives its acceleration as resulting from imbalance between the forces tugging at all four sides: it curves over in the *x and* the *y*-direction,

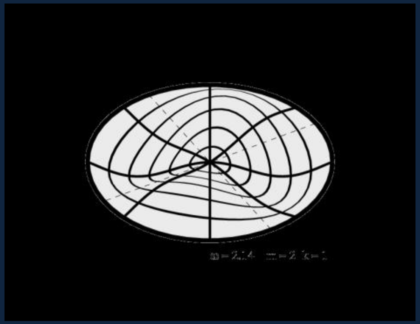
$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=\frac{\rho}{T}\frac{\partial^2 f}{\partial t^2}$$

Waves on a Drumhead

The two-dimensional wave equation can be solved to find the fundamental and harmonics of a vibrating drum head. Here are some of the modes of vibration (click to play):







These are from James Nearing, University of Miami

Different two-dimensional shapes have different boundary conditions, we can see different modes of vibration by forcing a node at a particular place in a vibrating system—the Chladni plates, for example, vibrating plates with sand on top. The sand comes to rest in the nodes, which are not points but curves.

Waves in Three Dimensions

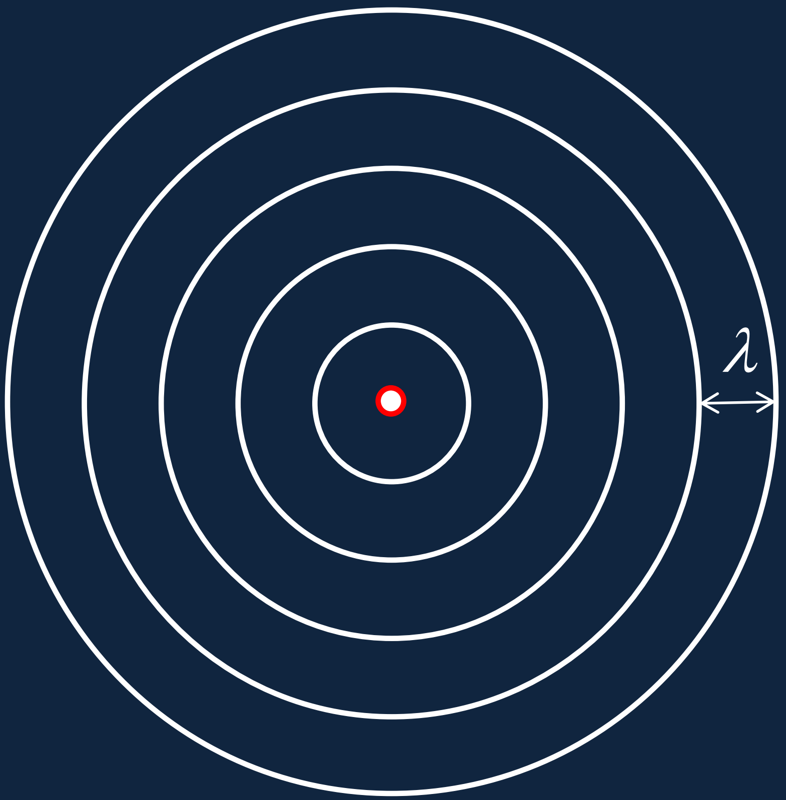
- The equation now is for a small cube being buffeted around by varying pressures on its six faces! The equation is:

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}+\frac{\partial^2 f}{\partial z^2}=\nabla^2 f=\frac{\rho}{B}\frac{\partial^2 f}{\partial t^2}$$

- This combination of differentiations comes up so often we have a special symbol, called del squared.
- This is the equation (for a component of local displacement, or for local density) that describes how sound waves get from me to you—it may look pretty scary, but don't worry, we won't need it except to know it works for harmonic waves going out spherically, and it's linear, so we can just add waves.

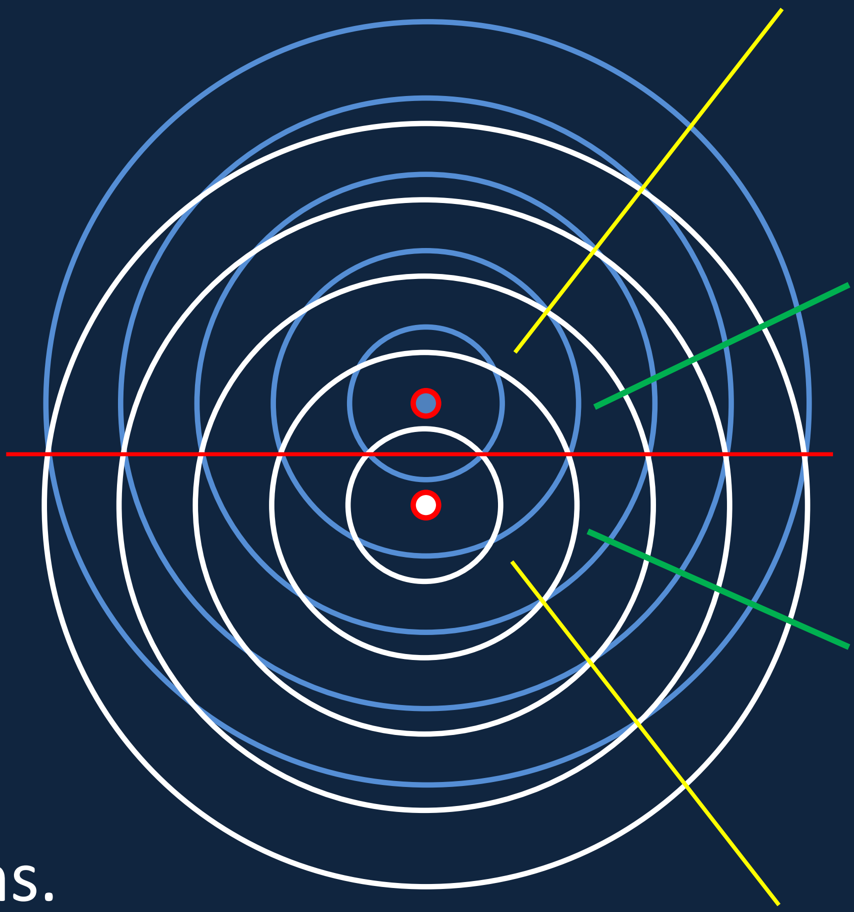
Sound Waves in Three Dimensions

- Think of a small source emitting a steady harmonic note: the equally spaced crests of the wave radiate outwards in concentric spheres, represented here by circles, so their radii are one wavelength λ apart.



Wave Interference

- Imagine now two such sources, emitting waves of the same wavelength in sync with each other.
- The air displacement at any point will be the vector sum of the two displacements (the waves add).
- On the red line, the crests add.
- Green line: crests add to troughs.
- Yellow line: crests add.

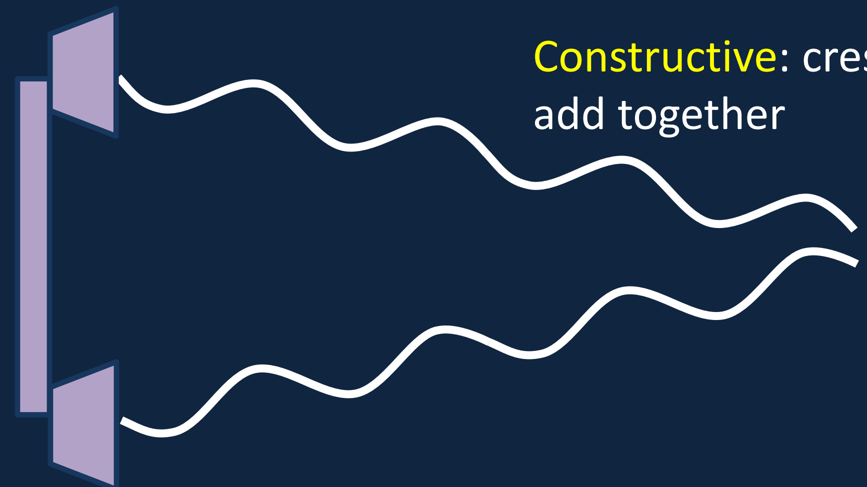


Green line: quiet zone, nodal line.
Red and Yellow: antinodal lines.

Excellent Website

Interference of Two Speakers

- Take two speakers producing in-phase harmonic sound.
- There will be constructive interference at any point where the difference in distance from the two speakers is a whole number of wavelengths $n\lambda$, destructive interference if it's an odd number of half wavelengths $(n + ½)\lambda$.

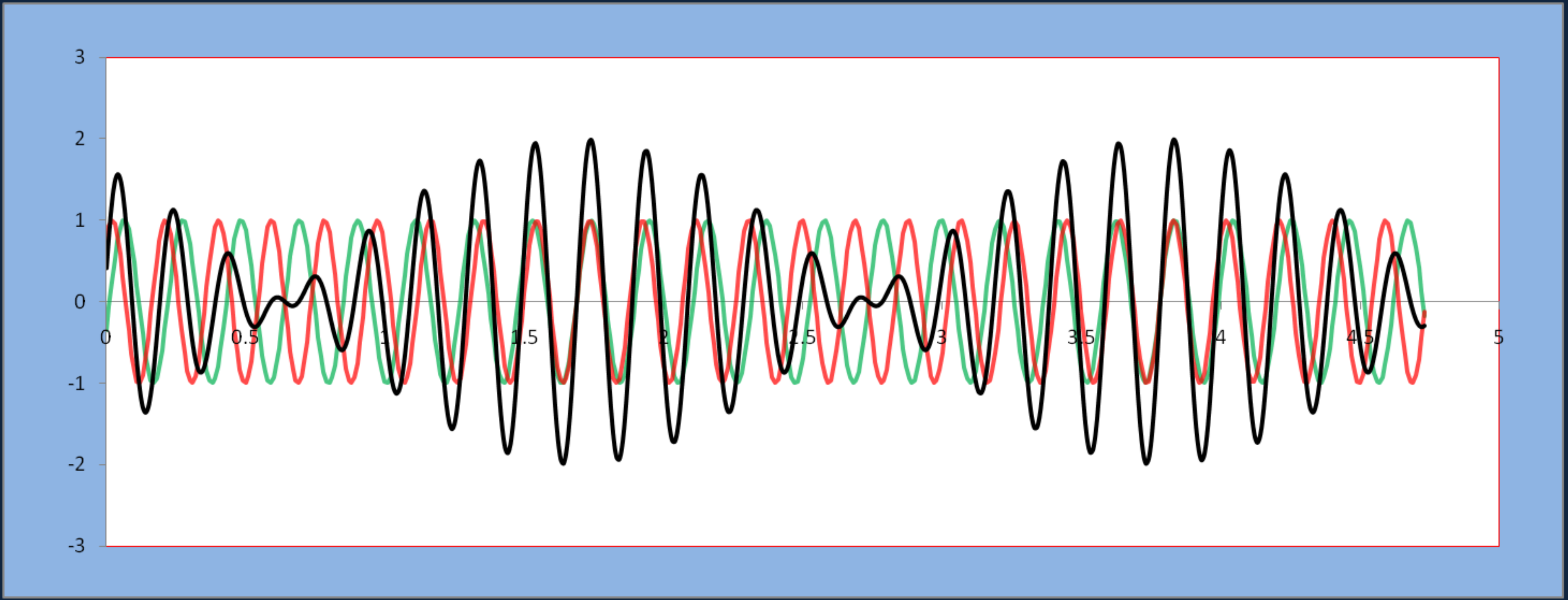


Constructive: crests add together

Destructive: crest meets trough, they annihilate

Beats

- If two harmonic waves close in frequency are added, they gradually go in and out of phase, the amplitude maxima (beats) occur with frequency equal to the difference of the two waves.



This is $k_1 = \omega_1 = 30$, $k_2 = \omega_2 = 33$.

Beats

- Adding the two harmonic waves:

$$A\sin\left(k_1 x-\omega_1 t\right)+A\sin\left(k_2 x-\omega_2 t\right)$$
$$=2\sin\left(\frac{k_1+k_2}{2}x-\frac{\omega_1+\omega_2}{2}t\right)\cos\left(\frac{k_1-k_2}{2}x-\frac{\omega_1-\omega_2}{2}t\right)$$

- The first sin term is a harmonic wave half way between the two being added, the cosine term is a slowly varying <u>modulation</u>: it has frequency equal to half the frequency difference of the two waves added, but beats occur twice per cycle, when cos has maximum amplitude, so at f_1-f_2.

The Doppler Effect

- For a harmonic source at rest, the crests are shown as circles separation λ where $\lambda f_0 = v$, the crests arrive with time interval $\tau_0 = 1/f_0$, note that $v\tau_0 = \lambda$.
- If source moves at speed u_s, between emitting crests it moves $u_s\tau_0$, so for crests moving to right, wavelength is shortened,

$$\lambda' = \lambda - u_s\tau_0$$

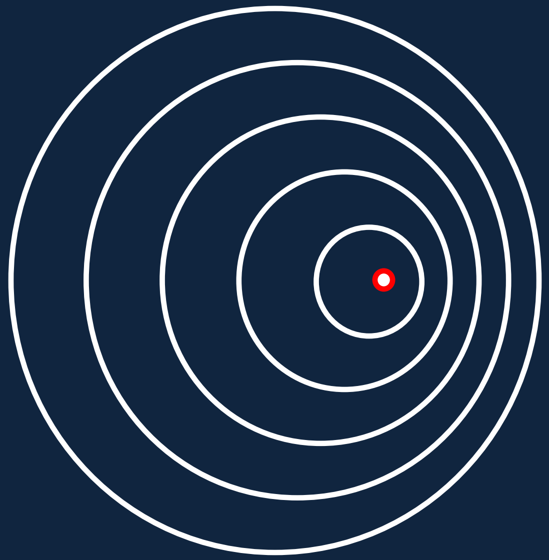
The Doppler Effect

- An observer to the right of the source will hear waves of wavelength $\lambda' = \lambda - u_s \tau_0$
  (τ_0 being the interval between crests being emitted)

  meaning he'll hear frequency

$$f' = \frac{v}{\lambda'} = \frac{v}{\lambda - u_s \tau_0}$$

$$= \frac{v}{\lambda}\left(\frac{1}{1 - u_s \tau_0 / \lambda}\right) = f_0\left(\frac{1}{1 - u_s / v}\right).$$



Left Behind!

- What about an observer to the left of the source?

  By an exactly similar argument, she'll hear a lower frequency,

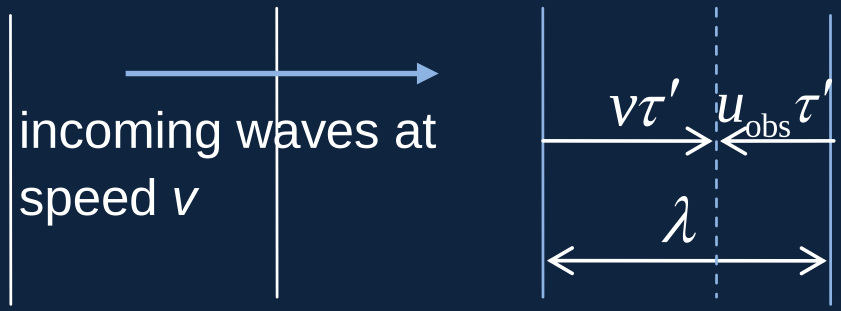
$$f' = f_0\left(\frac{1}{1+u_s/v}\right).$$

Stationary Source, Moving Observer

- If the observer is moving directly towards the stationary source, he will hear crests reaching him time τ' apart, where $(v+u_{\text{obs}})\tau' = \lambda = v\tau_0$,

  so

$$f' = \frac{1}{\tau'} = \frac{v+u_{\text{obs}}}{v\tau_0} = \left(1+\frac{u_{\text{obs}}}{v}\right)f$$



The observer moves at u_{obs} towards the incoming waves, meeting successive crests at time intervals τ'