# Waves I

Physics 2415 Lecture 25

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# Today’s Topics

- Dimensions
- Wave types: transverse and longitudinal
- Wave velocity using dimensions
- Harmonic waves

# Dimensions

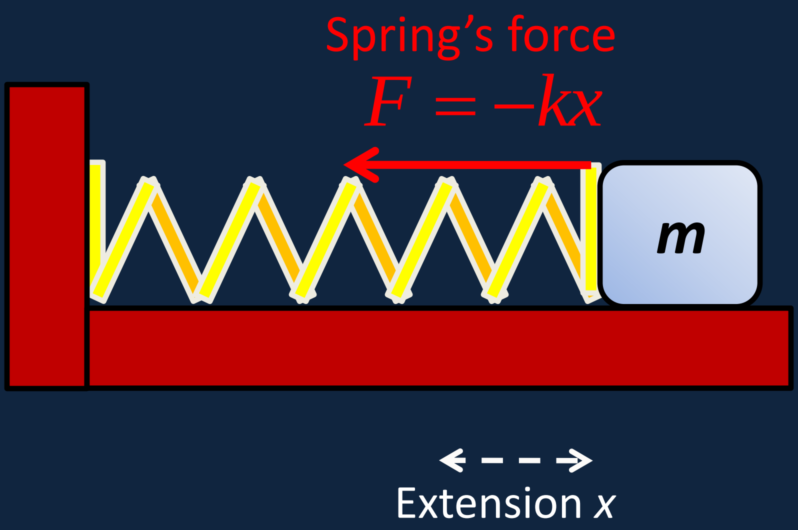
- There are three fundamental units in mechanics: those of mass, length and time.
- We denote the dimensions of these units by M, L and T.
- Acceleration has dimensions $LT^{-2}$ (as in $m/sec^2$, or mph per second—same for any unit system). Write this $[a] = LT^{-2}$.
- From $F = ma$, $[F] = [ma]$ so $[F] = MLT^{-2}$.

# Using Dimensions

- Example: period of a simple pendulum. What can it depend on?
- $[g] = LT^{-2}$, $[m] = m$, $[\ell] = L$.
- What combination of these variables has dimension just T? No place to include $m$, and we need to combine the others to eliminate L:
- $[g/\ell] = T^{-2}$, so $\sqrt{\ell / g}$ is the only possible choice.
- Dimensional analysis can't (of course) give dimensionless factors like $2\pi$.

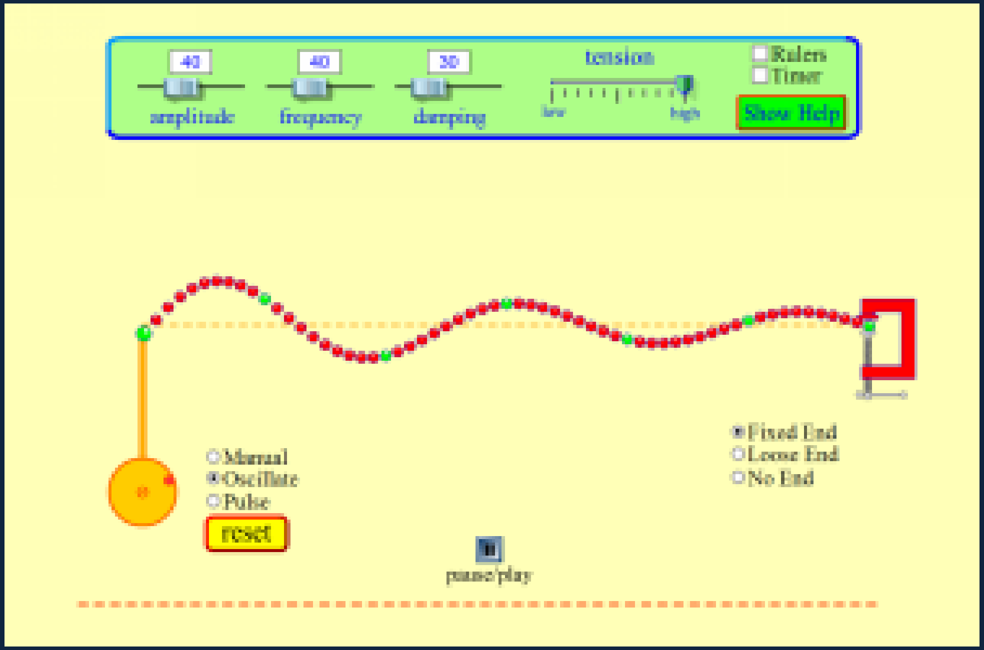
# Dimensional Analysis: Mass on Spring

- From $F = -kx$,
  $[k] = [F]/[x] = \text{MLT}^{-2}/\text{L} = \text{MT}^{-2}$.
- How does the period of oscillation depend on the spring constant $k$?
- The period has dimension T, the only variables we have are $k$ and $m$, the only combination that gives dimension T is $\sqrt{m/k}$, so we conclude that $T \propto 1/\sqrt{k}$ .



# Waves on a String

*A simulation from the University of Colorado*

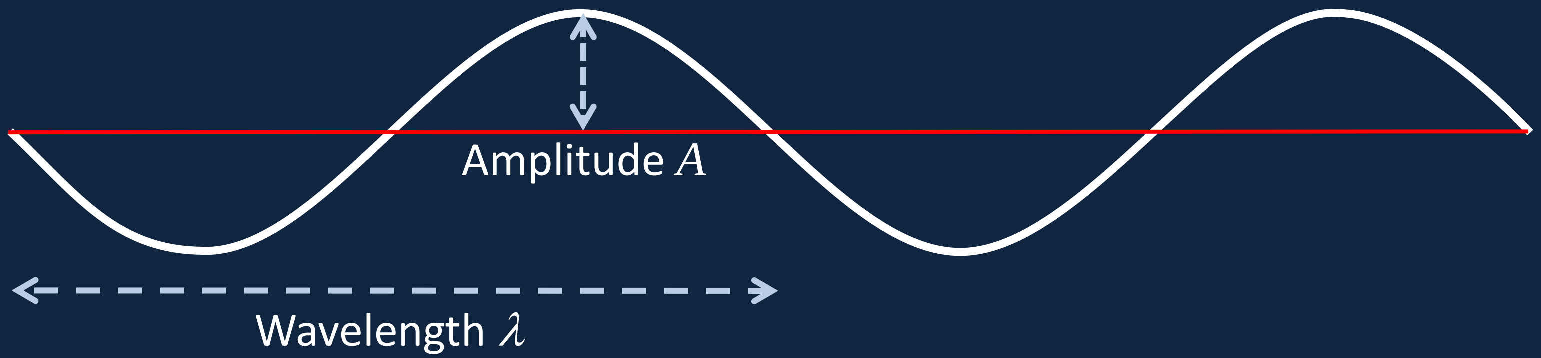


# Transverse and Longitudinal Waves

- The waves we've looked at on a taut string are transverse waves: notice the particles of string move up and down, perpendicular to the direction of progress of the wave.
- In a longitudinal wave, the particle motion is back and forth along the direction of the wave: an example is a sound wave in air.

# Harmonic Waves

- A simple harmonic wave has sinusoidal form:




- For a string along the *x*-axis, this is local displacement in *y*-direction at some instant.
- For a <u>sound wave</u> traveling in the *x*-direction, this is local *x*-displacement at some instant.

# Wave Velocity for String

- The wave velocity depends on string tension $T$, a force, having dimensions $MLT^{-2}$, and its mass per unit length $\mu$ , dimensions $ML^{-1}$.
- What combination of $MLT^{-2}$ and $ML^{-1}$ has dimensions of velocity, $LT^{-1}$?
- We get rid of M by dividing one by the other, and find $[T/\mu] = L^2T^{-2}$ :
- In fact, $v = \sqrt{T / \mu}$ is <u>exactly correct</u>!
- This is partly luck—there could be a dimensionless factor, like the $2\pi$ for a pendulum.

# Sound Wave Velocity in Air

- Sound waves in air are pressure waves. The obvious variables for dimensional analysis are the pressure $[P]$ = [force/area] = $MLT^{-2}/L^2$ = $ML^{-1}T^{-2}$ and density $[\rho]$ = [mass/vol] = $ML^{-3}$.
- Clearly $\sqrt{P/\rho}$ has the right dimensions, but detailed analysis proves

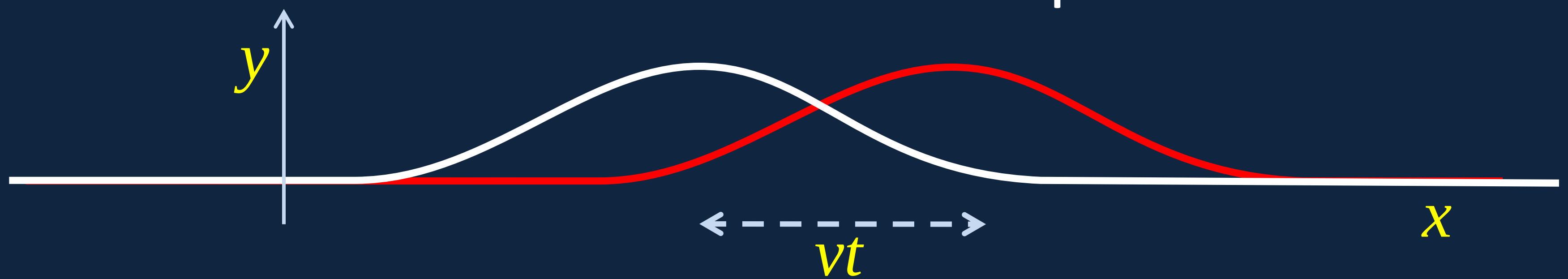
$$v = \sqrt{\partial P/\partial\rho} = \sqrt{\gamma P/\rho}$$

where $\gamma = 1.4$.

- This can also be written in terms of the bulk modulus $B = \rho\left(\partial P/\partial\rho\right)$, but that differentiation must be adiabatic—local heat generated by sound wave pressure has no time to spread, this isn't isothermal.

# Traveling Wave

- Experimentally, a pulse traveling down a string under tension maintains its shape:




- Mathematically, this means the perpendicular displacement $y$ stays the same function of $x$, but with an origin moving at velocity $v$:

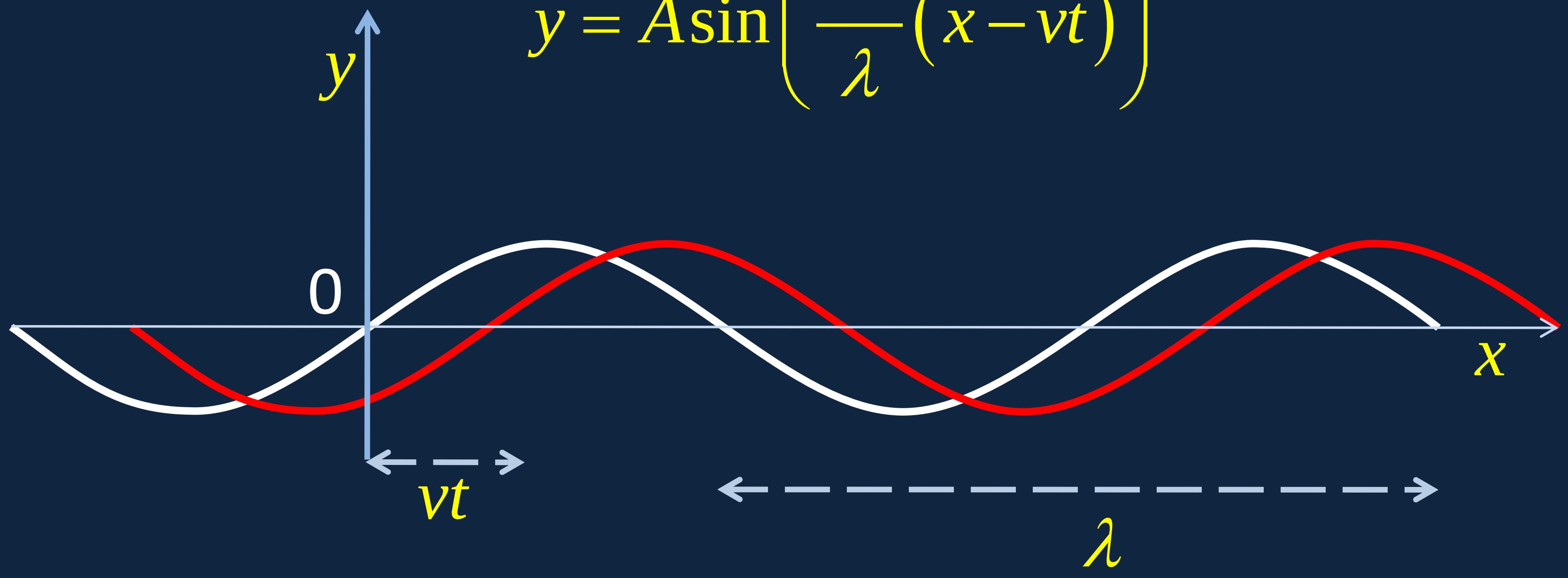
$$y = f(x,t) = f(x - vt)$$

*So the white curve is the physical position of the string at time zero, the red curve is its position at later time $t$.*

# Traveling Harmonic Wave

- A sine wave of wavelength $\lambda$, amplitude $A$, traveling at velocity $v$ has displacement

$$y = A\sin\left(\frac{2\pi}{\lambda}(x - vt)\right)$$



# Harmonic Wave Notation

- A sine wave of wavelength $\lambda$, amplitude $A$, traveling at velocity $v$ has displacement

$$y = A\sin\left(\frac{2\pi}{\lambda}(x - vt)\right)$$

- This is usually written $y = A\sin(kx - \omega t)$, where the "wave number" $k = 2\pi/\lambda$ and $\omega = vk$.
- As the wave is passing, a single particle of string has simple harmonic motion with frequency $\omega$ radians/sec, or $f = \omega/2\pi$ Hz. Note that $\boxed{v = \lambda f}$