Faraday's Law of Induction III

Physics 2415 Lecture 21

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Today’s Topics

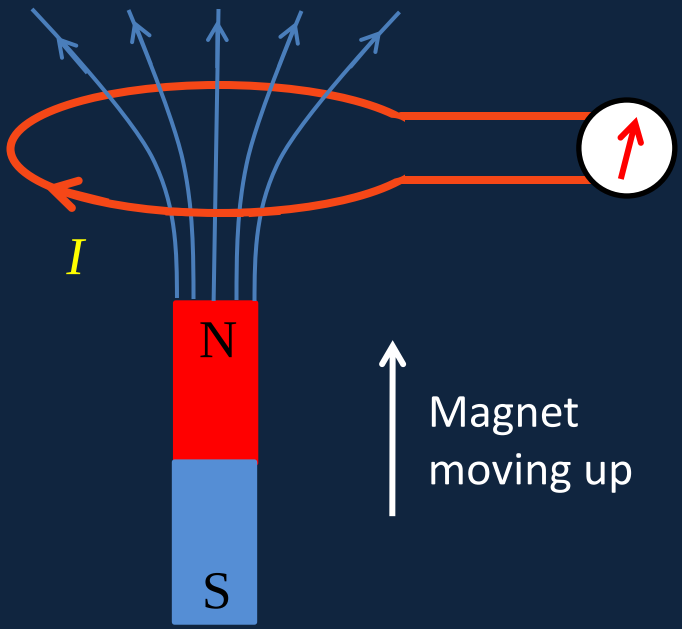
- More on Faraday’s Law of Induction
- Generators
- Back emf and Counter Torque
- Transformers
- General form of Faraday’s Law

Faraday's Law of Induction

- Faraday's law of induction states that when the magnetic flux through a loop is changing, there is an induced emf in the loop given by:

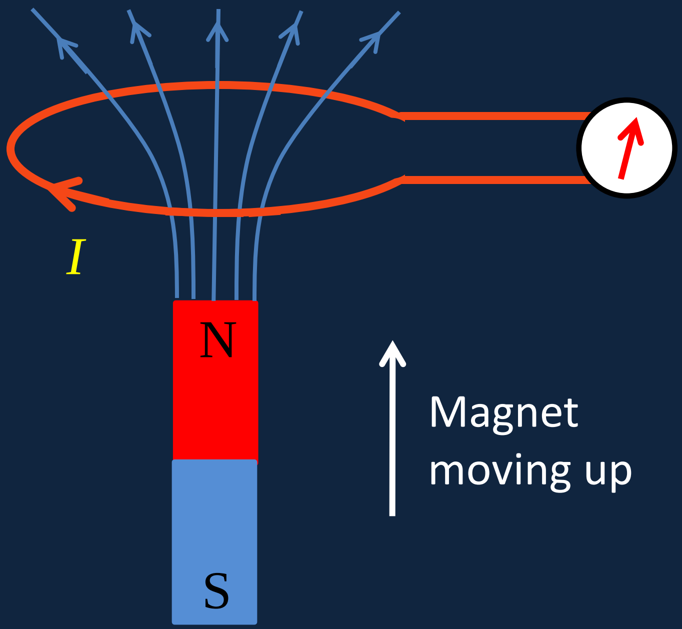
$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

- You get the sign of the emf from Lenz's law…



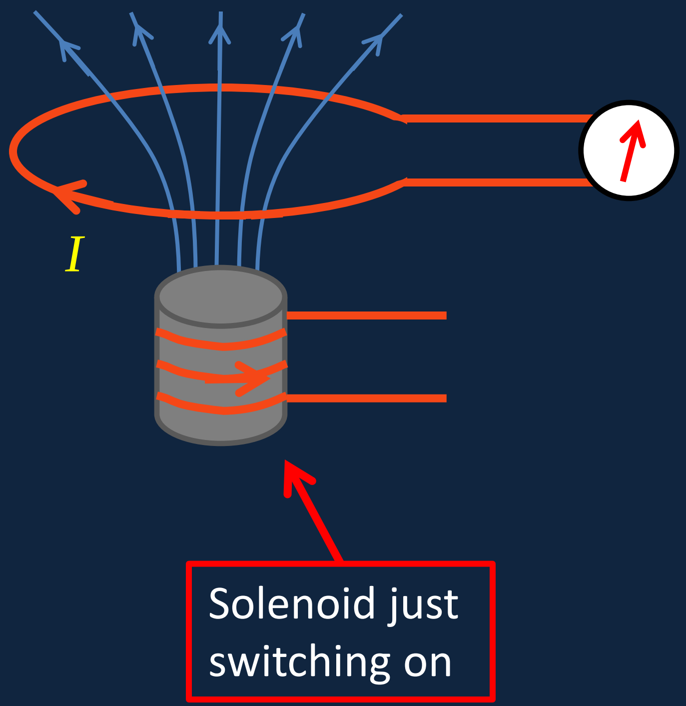
Lenz's Law

- The direction of the induced emf generated by a changing magnetic flux is always such as to oppose the motion.
- Example: as the N pole moves up towards the loop, the current induced generates an N pole underneath to repel and slow down the approaching magnet.



Lenz's Law Continued…

- The direction of the induced emf generated by a changing magnetic flux is always such as to oppose the change in flux through the loop.
- Example: as the solenoid switches on, creating upward magnetic flux through the loop, the current generated in the loop will add downward flux.

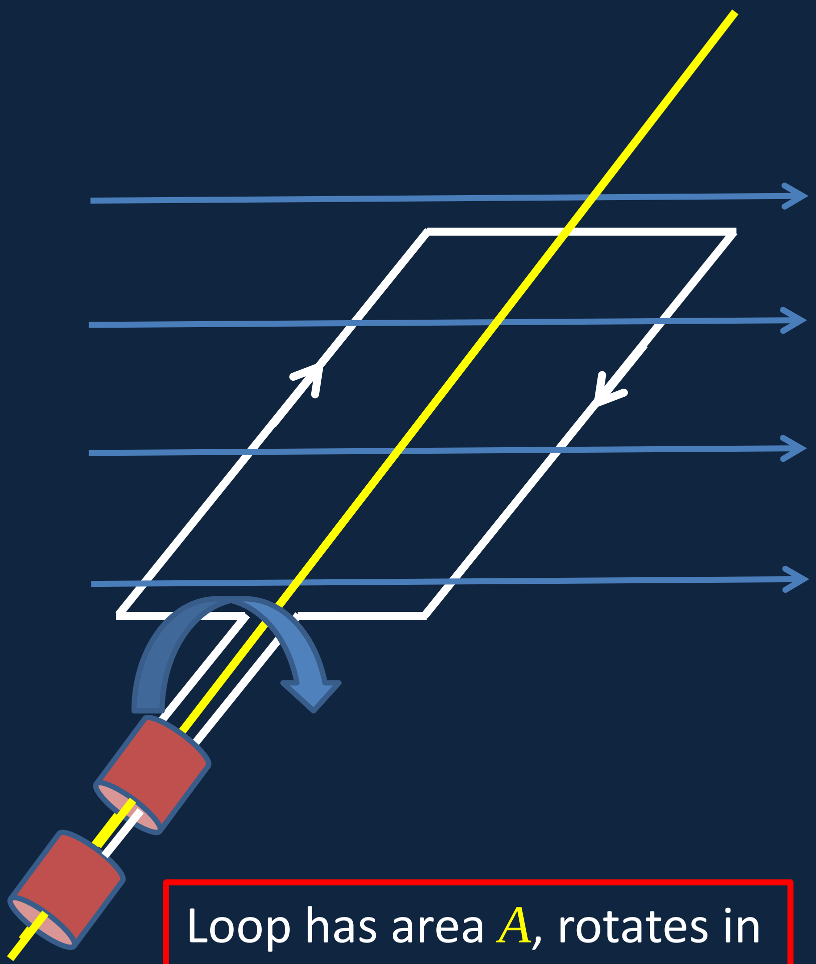


AC Electric Generators

- The essential mechanism is a loop, or in practice a coil of many loops, rotating in a magnetic field, such as between the poles of a horseshoe magnet.
- If the current is collected via slip rings (no commutator) it will be ac, for one loop:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt} BA\cos\omega t = BA\omega\sin\omega t$$

ωt is the angle between $\vec{B}$ and coil area vector $\vec{A}$

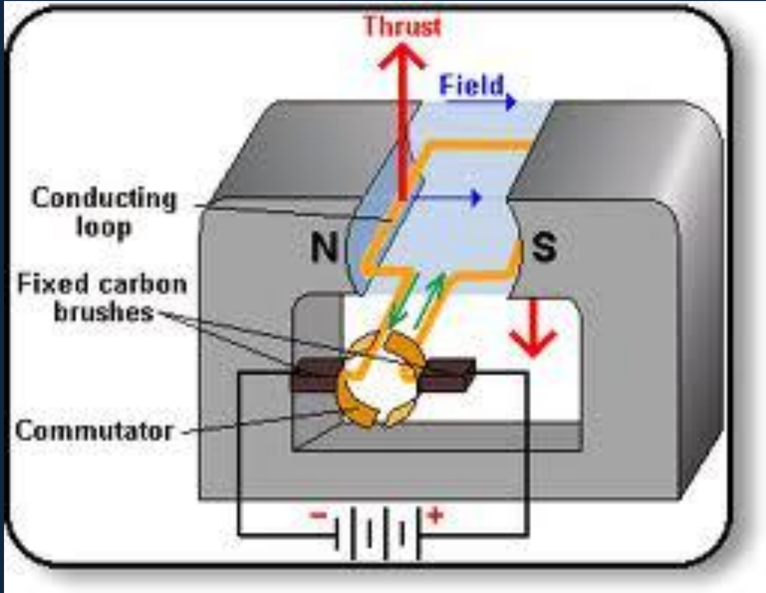


Loop has area A, rotates in field B at ω radians/sec.

Electric Motor and Back emf

Animation!

- As the loop rotates (think of it as a short bar magnet attracted towards the poles of the big magnet) the commutator switches current direction, and therefore switches the loop's poles.



But now we see that as the loop rotates in the magnetic field, that rotation will induce an emf in the loop opposing the motion—in other words, opposing the driving emf!
This is called back emf, and is proportional to speed.

More about back emf…

- When a motor is first connected, it is not turning and Ohm's law gives $V_0 = IR$, where V_0 is the voltage of the supply, and R the resistance of the armature (meaning the loop or coil). Heat production inside the motor is I^2R.
- When the motor is running under load, there is a back emf V_{back}, and now $V_0 - V_{\text{back}} = I\,R$.
- Heat production in the motor is now $I^2\,R$: which can be much less than initially!
- If a blender is mechanically overloaded so the motor turns slowly, back emf is small, the current is higher than designed for, high heat production for some time may cause burnout.

Back emf problem

- A motor has an armature resistance of 4Ω.
- It draws 10A from a 120-V line when running at its design speed of 1000 rpm.
- If a load slows it to 250 rpm, what is the current in the armature?

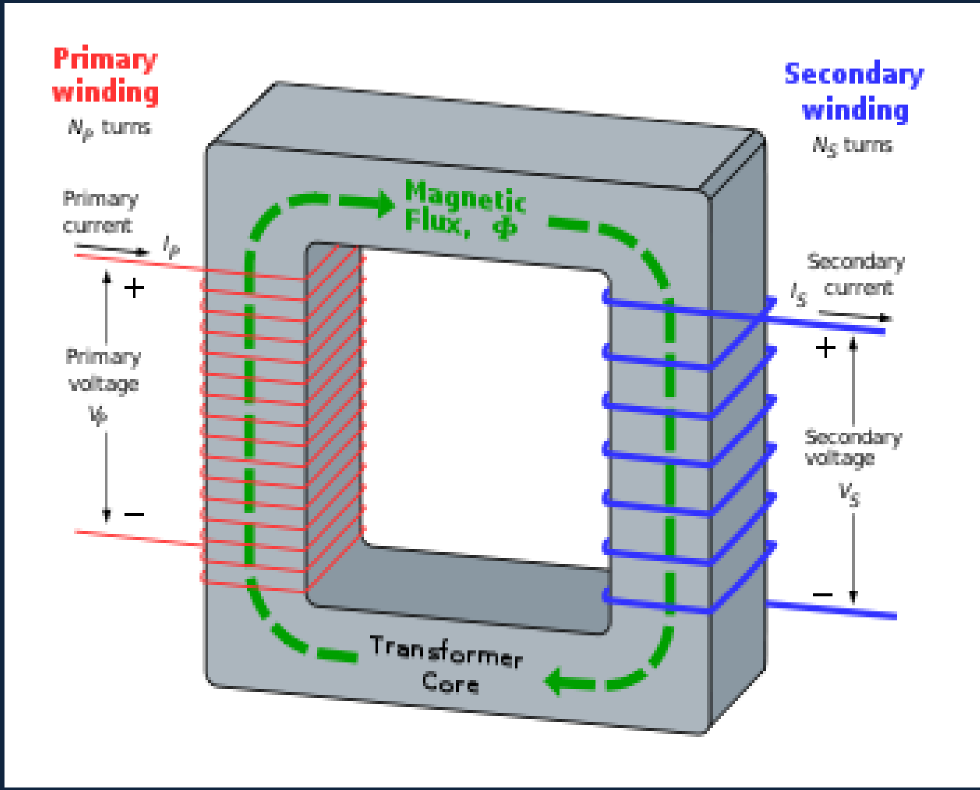
Counter Torque

- A generator is essentially a loop rotating in a magnetic field.
- If the generator is connected to an outside circuit, the induced emf will cause a current to flow: that's the point of the generator!
- But the current carrying wire moving through the field will feel Lenz-type forces opposing its motion: called the "counter torque".
- So to produce a current through the external circuit work must be done. Obviously!

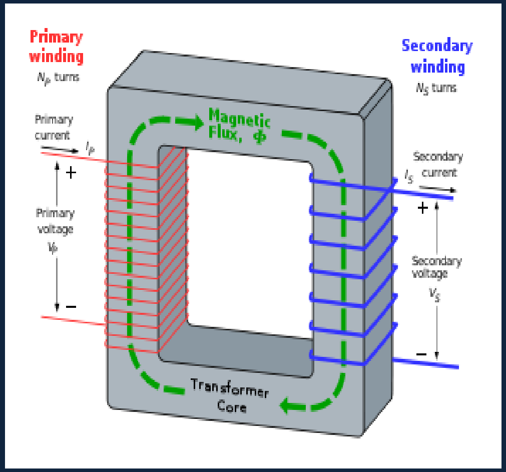
Demo: bicycle driving generator

Transformers

- AC power is easy to change to a different voltage.
- An AC source feeds the primary coil. Ideally, all the magnetic field produced lies in the iron ring and passes also through the secondary coil, so both coils have the same flux per loop.



Transformers



- If at some instant the flux in the iron loop is Φ_B, passing through the N_S loops of the secondary coil and the N_P loops of the primary, the voltages in the two coils are

$$V_S = N_S d\Phi_B / dt, \quad V_P = N_P d\Phi_B / dt$$

from which

$$\boxed{V_S / V_P = N_S / N_P}$$

The Perfect Transformer

- Most transformers are extremely efficient, above 99%, so this is a good approximation.
- Negligible resistance and eddy currents.
- The voltage V_P supplied to the primary is balanced exactly by the "back emf" generated by the changing magnetic field. (If the secondary is part of a circuit, it's contributing to this field too.)
- Power in = power out: $I_P V_P = I_S V_S$

Problem from book

- A model-train transformer plugs into 120 V ac and draws 0.35A while supplying 7.0A to the train.
- What voltage is present across the tracks?
- Is the transformer step-up or step-down?

Faraday's Law: General Form

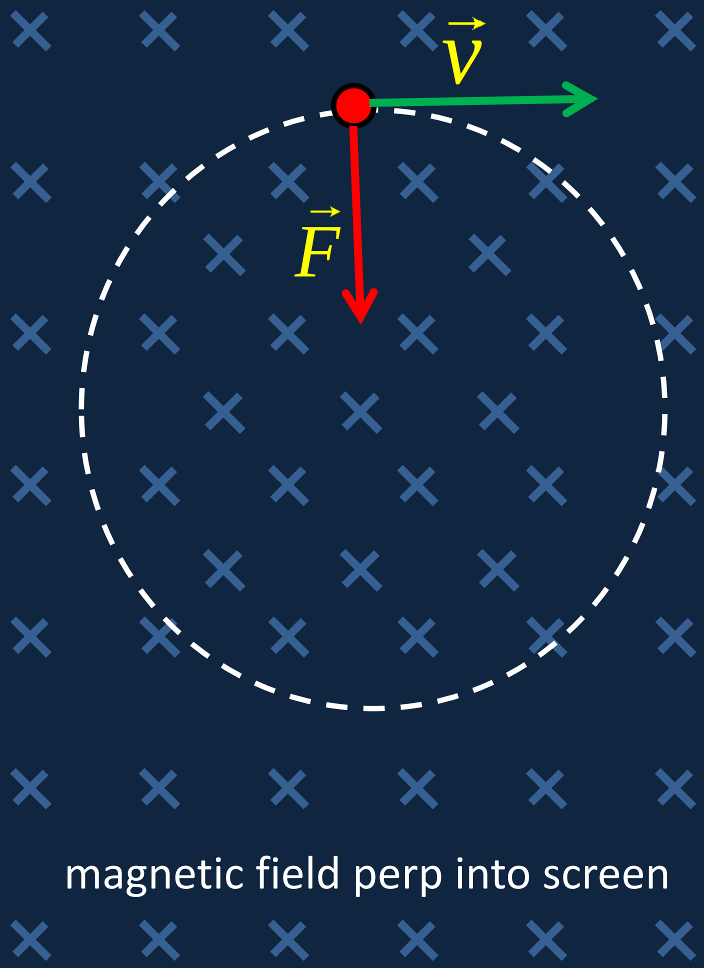
- A changing magnetic flux through a loop generates an emf around the loop which will drive a current. The emf can be written:

$$\mathscr{E} = \oint_{\text{loop}} \vec{E} \cdot \overrightarrow{d\ell} = -\frac{d\Phi_B}{dt}$$

In fact, this electric field is there even without the wire: if an electron is circling in a magnetic field, and the field strength is increased, the electron accelerates, driven by the circling electric field—the basis of the betatron.

The Betatron

- If an electron is circling in a magnetic field, and the magnetic field intensity is increased, from Faraday's law there will be circling lines of electric field which accelerate the electron. It is easy to design the field so that the electron circles at constant radius—electrons can attain 99.9% of the speed of light this way.



magnetic field perp into screen

A betatron was used as a trigger in an early nuclear bomb.