Sources of Magnetic Field I

Physics 2415 Lecture 17

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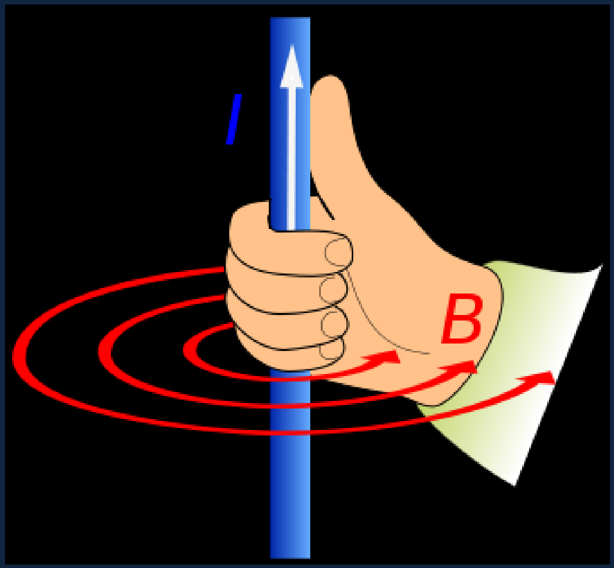
Today’s Topics

- Forces between currents
- Ampère’s Law
- Fields inside wire and solenoid

Magnetic Field from a Current in a Long Straight Wire

- The lines of magnetic force are circles around the wire, direction determined by the right hand rule.
- From experiment, the field strength is proportional to the current, and *inversely* proportional to distance from the wire:



Animation

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}, \quad \mu_0 = 4\pi \times 10^{-7}\,\text{Tesla} \cdot \text{m/A}$$

The 2π is put in to make some later formulas simpler!

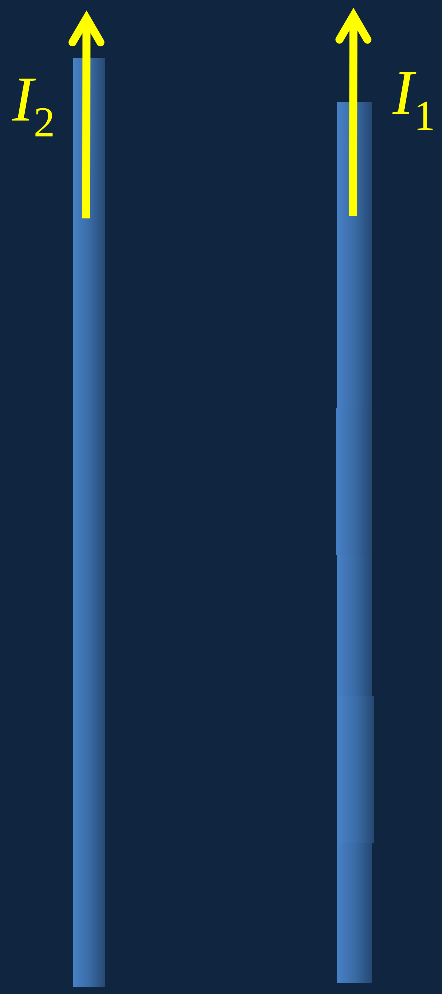
Clicker Question

- Currents flow in the same direction in parallel wires.
- Do the wires

A. Repel each other?

B. Attract each other?

C. Neither attract nor repel?



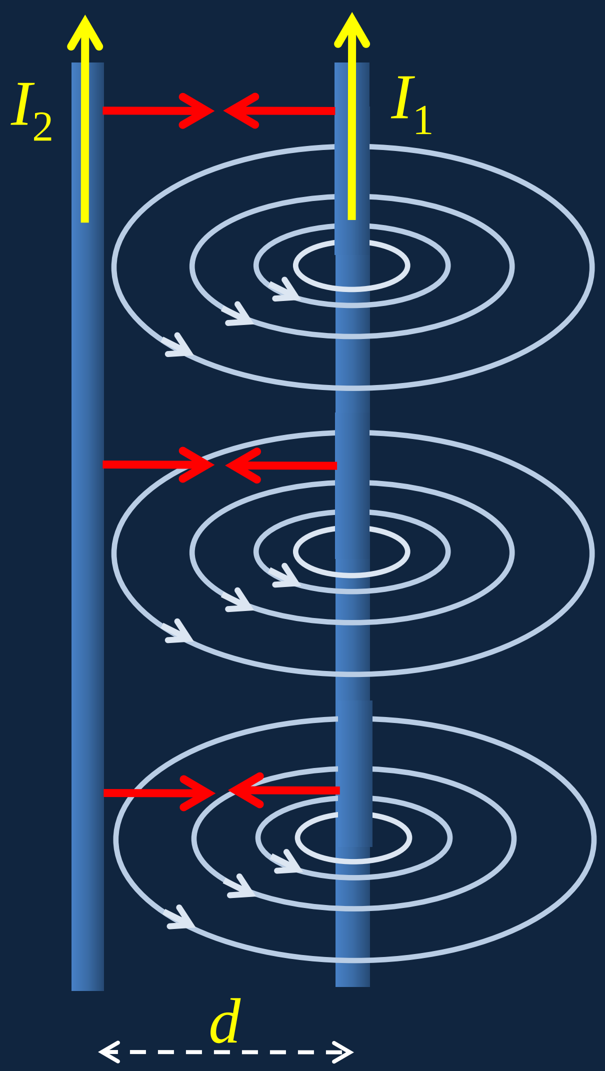
Force Between Parallel Wires

- The field from current I_1 is $B = \frac{\mu_0}{2\pi}\frac{I_1}{r}$, circling the wire, and the current I_2 will feel a force $I_2 \vec{\ell} \times \vec{B}$ per length ℓ, so the force per meter on wire 2 is

$$F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$$

towards wire 1, and wire 1 will feel the opposite force.

- <u>Like currents attract.</u>



Definition of the Ampère and Coulomb

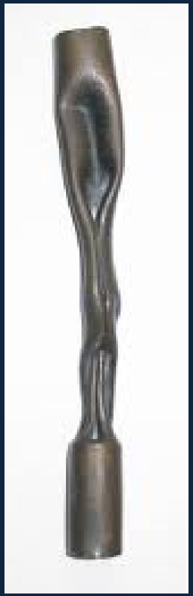
- In the formula for the attraction between long parallel wires carrying steady currents

$$F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$$

the constant μ_0 has <u>precisely</u> the value $4\pi \times 10^{-7}$.

- Fixing μ_0 <u>defines</u> the unit of current, the ampère, as that current which in a long wire one meter away from an equal current feels a force of $\mu_0/2\pi$ N/m—and 1 amp = 1 coulomb/sec.

Like Currents Attracting

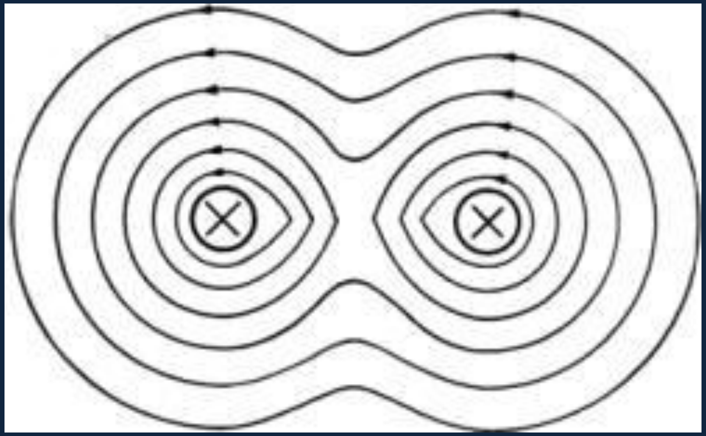


- The picture on the left is of a copper pipe used as a lightning conductor—after it conducted. The parallel currents all attracted each other.
- On the right, an intense current is sent through a plasma—the self compression generates intense heat. The hope is to induce thermonuclear fusion.

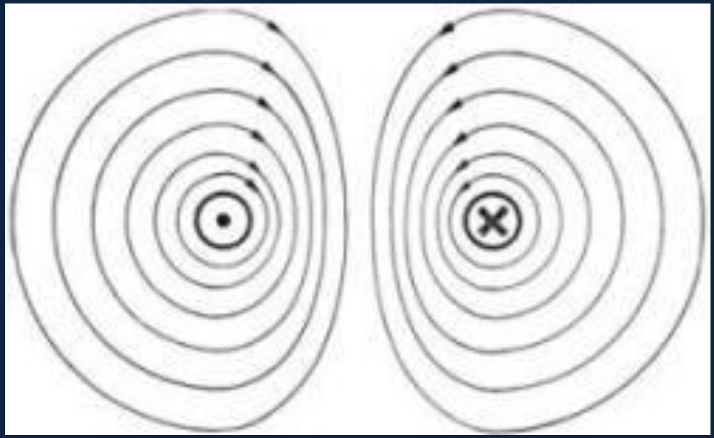
Magnetic Field Lines for Parallel Wires

- The magnetic field at a point is the vector sum of the two fields circling the wires.
- For equal magnitude currents, the field lines are as shown.
- Faraday visualized the lines as elastic, trying to minimize their length, and also repelling each other sideways. This helps see how likes attract, opposite currents repel.

Interactive animation here



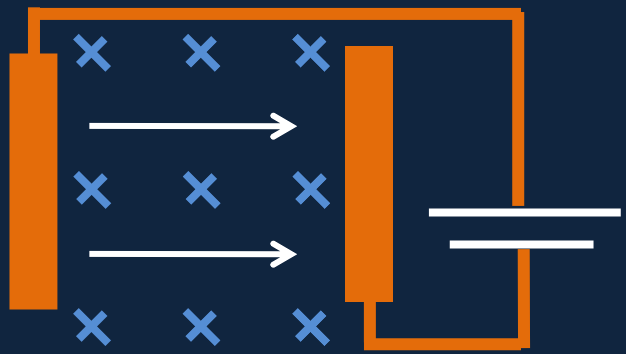
Currents in same direction



Currents in opposite direction

Magnetohydrodynamic Drive

- Seawater conducts electricity: the idea behind the Red October silent drive was that an electric current through seawater, with a perpendicular magnetic field, would drive the water in the direction perpendicular to both, moving the water with no vibration…but the fluid flow has stability problems, little progress so far...



Clicker Question

- How long is the stretch of circumference between two radii 0.1 radians apart, if $r = 2$?

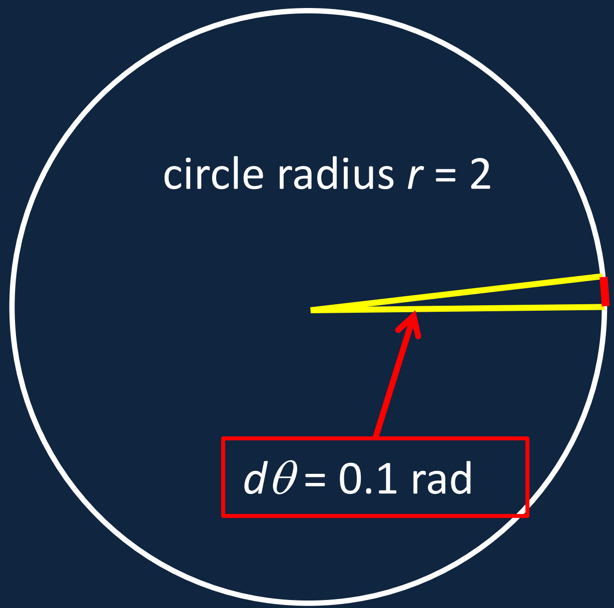
A. 0.2

B. 0.4

C. 0.1

D. $0.1/\pi$

E. 0.1π

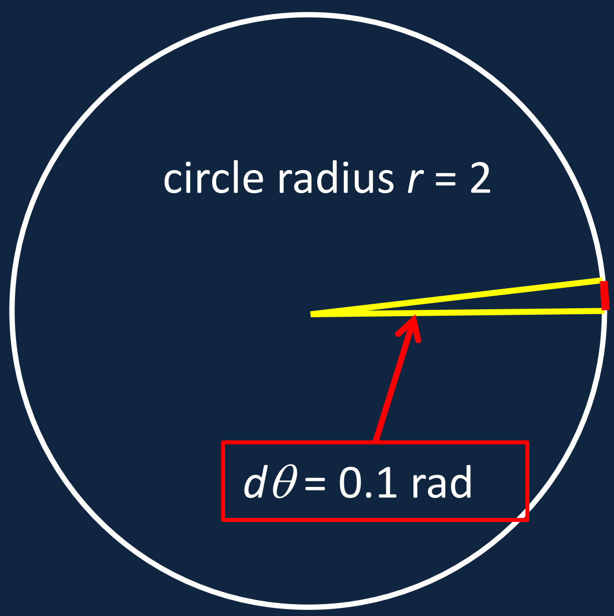


Clicker Answer

- How long is the stretch of circumference between two radii 0.1 radians apart, if $r = 2$?

A. 0.2

- For a circle of radius r, the length of circumference corresponding to an angle θ radians is $r\theta$.
- Remember 360° = 2π radians, and all the way round is $2\pi r$.

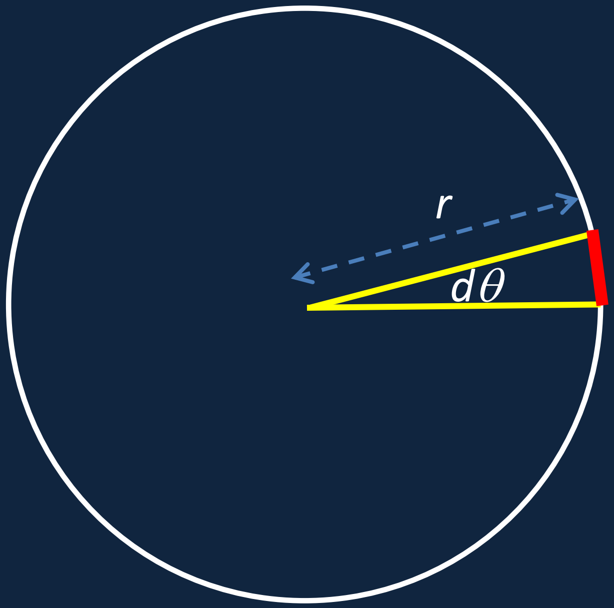


Introducing Ampère's Law

- Suppose we have an infinite straight wire with current I coming perpendicularly out of the screen at the center of the circle. Then we know that the magnetic field from the current has circular lines, and strength

$$B = (\mu_0 / 2\pi)(I / r).$$

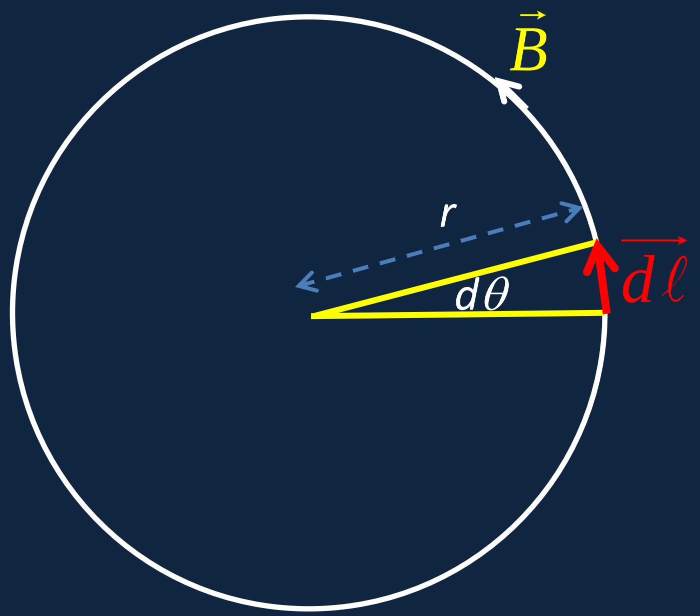
- What is the value of the integral around a circle $\oint \vec{B} \cdot \overrightarrow{d\ell}$?



Introducing Ampère's Law

- Current I coming out of page at center, $B = (\mu_0 / 2\pi)(I / r)$.
- What is the value of the integral around a circle $\oint \vec{B} \cdot \vec{d\ell}$?
- From the red bit of the circle, $d\ell = rd\theta$, and $\vec{B} \parallel \vec{d\ell}$ so

$$\vec{B} \cdot \vec{d\ell} = (\mu_0 / 2\pi)(I / r) rd\theta$$
$$= (\mu_0 / 2\pi) Id\theta$$



- from which

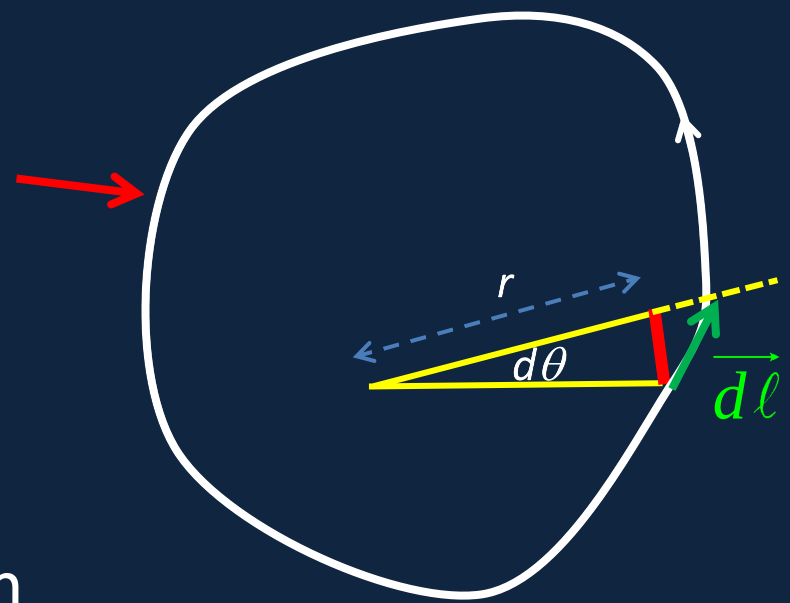
$$\oint \vec{B} \cdot \vec{d\ell} = (\mu_0 / 2\pi) I \oint d\theta = \mu_0 I$$

- This is Ampère's Law.

Ampère's Law: General Path of Integration

$$B = (\mu_0 / 2\pi)(I / r)$$

Now take *noncircular* path shown:



- We're still finding $\oint \vec{B} \cdot \overrightarrow{d\ell}$
- Look at the little green $\overrightarrow{d\ell}$:
- $\vec{B}$ circles the wire, so $\vec{B}$ is in the direction of the red bit, and again

$$\vec{B} \cdot \overrightarrow{d\ell} = (\mu_0 / 2\pi)(I / r) r d\theta$$
$$= (\mu_0 / 2\pi) I d\theta$$

$$\oint \vec{B} \cdot \overrightarrow{d\ell} = \mu_0 I$$

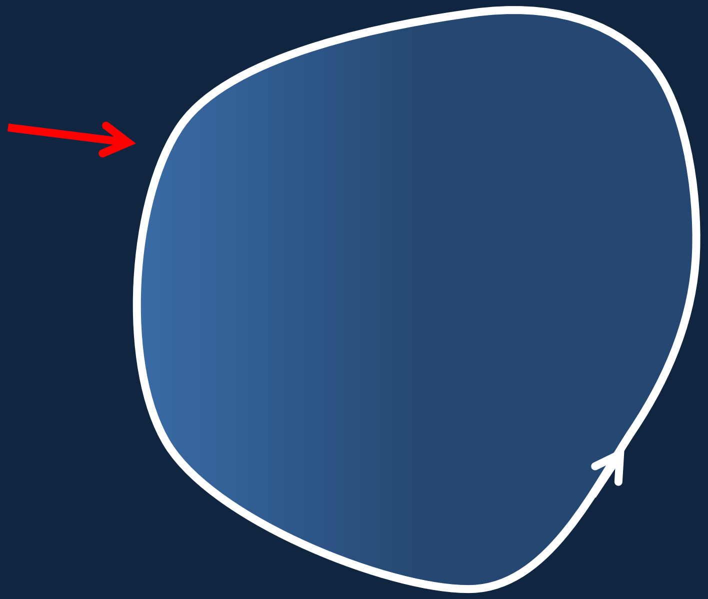
Important!
The component of $\overrightarrow{d\ell}$ in the direction of $\vec{B}$ has magnitude $r d\theta$

Ampère’s Law: General Case

- Ampère’s Law states that for any magnetic field generated by a steady flow of electrical currents, if we take an arbitrary closed path in space and integrate around it, then

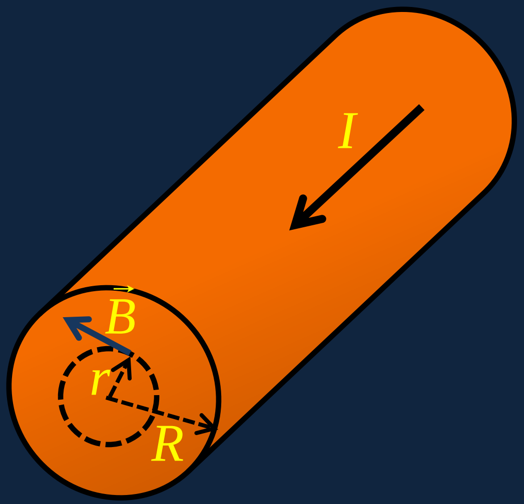
$$\oint \vec{B} \cdot \overrightarrow{d\ell} = \mu_0 I$$

where now I is the total net current flowing across any surface having the path of integration as its boundary, such as the blue surface shown here.



Field Inside a Wire

- Assume a wire of circular cross section, radius R, carries a current I uniformly distributed through its volume.
- By symmetry, the field inside must circle around—there can be no component pointing outwards or inwards, that would imply a single magnetic pole at the center of the wire.

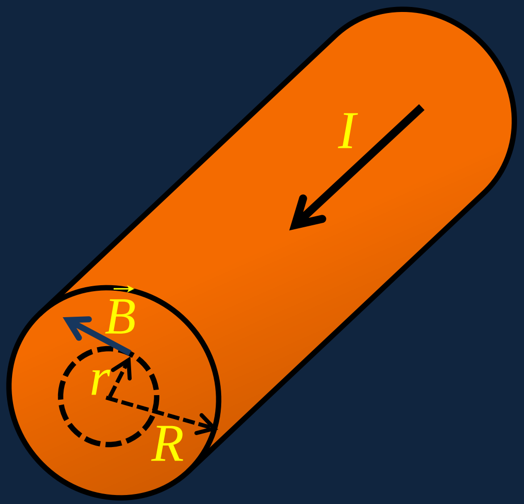


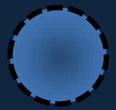
Field Inside a Wire

- Apply Ampère's law to the dotted path of radius r inside the wire as shown.
- The surface "roofing" this path has area πr^2, the whole wire has cross-section area πR^2, so the current flowing through the path is $I(r^2/R^2)$, and Ampère's law gives

$$\oint \vec{B} \cdot \overrightarrow{d\ell} = 2\pi r B = \mu_0 I \left(r^2 / R^2 \right),$$

$$B = \frac{\mu_0 I}{2\pi} \frac{r}{R^2}$$



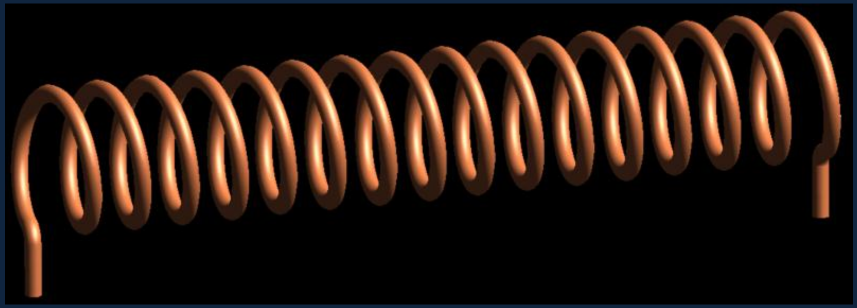


Magnetic Field Inside a Solenoid

- Take a rectangular Ampèrian path as shown. Assume the external magnetic field negligible, and the field inside parallel to the axis (a good approximation for a long solenoid). For current I, n turns/meter,

$$\oint \vec{B} \cdot \overrightarrow{d\ell} = B\ell = \mu_0 n \ell I$$

$$\boxed{B = \mu_0 n I}$$



rectangular path of integration