

# Electric Currents and Resistance II

Physics 2415 Lecture 11

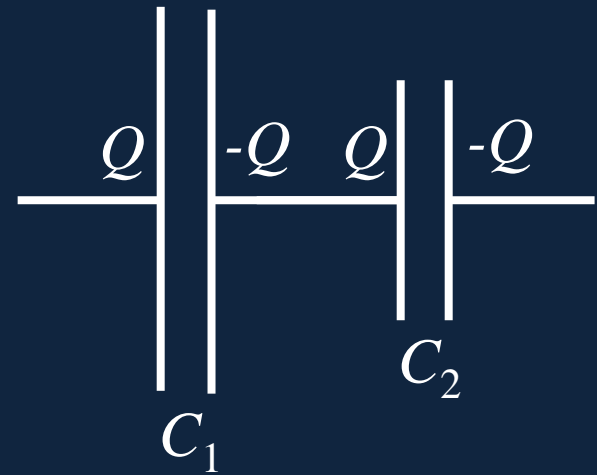
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# Today's Topics

- First we'll mention capacitors
- Power usage: kWh, etc.
- The microscopic picture
- Temperature dependence of resistivity
- Drift speed and electron speed
- AC and DC
- Semiconductors and superconductors

# Know This...

- Capacitors in parallel (any number) are all at the same voltage  $V$ .
- Capacitors in series (any number) all carry the same charge  $Q$ .
- Putting **these facts** together with  $V = Q/C$  can solve a lot of problems!



# Resistance and Resistivity

- To summarize: for a given material (say, copper) the resistance of a piece of uniform wire is proportional to its length  $\ell$  and inversely proportional to its cross-sectional area  $A$ .

- This is written:  $R = \rho \frac{\ell}{A}$

where  $\rho$  is the resistivity.

- For copper,  $\rho = 1.68 \times 10^{-8} \Omega \cdot \text{m}$ .

# Electric Power

- Remember voltage is a measure of potential energy of electric charge, and if one coulomb drops through a potential difference of one volt it loses one joule of potential energy.
- So a current of  $I$  amps flowing through a wire with  $V$  volts potential difference between the ends is losing  $IV$  joules per sec.
- This energy appears as heat in the wire: the electric field accelerates the electrons, which then bump into impurities and defects in the wire, and are slowed down to begin accelerating again, like a sloping pinball machine.

# Power and Energy Usage

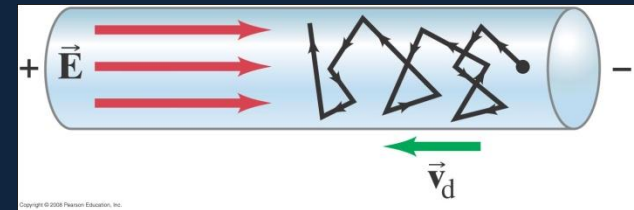
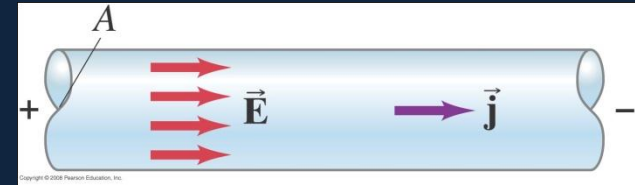
- Using Ohm's law, we can write the power use of a resistive heater (or equivalent device, such as a bulb) in different ways:

$$P = IV = I^2R = V^2 / R$$

- The unit is **watts**, meaning **joules per second**.
- Electric meters measure **total energy** usage: adding up how much power is drawn for how long, the standard unit is the kilowatt hour:
- **1 kWh = 1,000x3,600J = 3.6MJ**

# Microscopic Picture of Conductivity

- The total current down the wire is  $I$ ; if we assume it's uniform over the cross section area  $A$  (which it is) there is a **current density**  $j = I/A$ . (units: amps/m<sup>2</sup>)
- A constant  $E$  field gives a steady current. This means the electrons are bouncing off things, like a sloping pinball machine, otherwise the current would keep accelerating.



# What are the Electrons Bouncing off?

- Not the atoms! It's found experimentally that electrons pass dozens or often hundreds of atoms before being deflected.
- Furthermore, an extremely pure crystal of copper has a very low resistance if it's really cooled down....and the atoms are all still there.



# What are the Electrons Bouncing off?

- Not the atoms!
- An extremely pure crystal of copper has a very low resistance if it's really cooled down....
- This is the clue: they are deflected by **thermal vibrations of the lattice—resistance increases with temperature.**
- The electrons also bounce off impurities, but can pass through a pure cold lattice like light through glass... electrons are really waves!

# Temperature Dependence of Resistivity

- Resistivity of metals increases approximately linearly with temperature over a wide range.
- The formula is:

$$\rho_T = \rho_0 [1 + \alpha (T - T_0)]$$


$\rho_0$  being the resistance at some fixed  $T_0$ , and  $\alpha$  the temperature coefficient of resistivity.

- An ordinary incandescent bulb has a tungsten wire at about 3300K, and  $\alpha = 0.0045$ , from which  $\rho_T \approx 15\rho_0$  not so far off proportional to absolute temperature.

# Clicker Question

- What is the resistance of a 12V, 36 Watt headlight bulb?
  - A. 3 ohms
  - B. 4 ohms
  - C. 0.3 ohms


# Clicker Answer

- What is the resistance of a 12V, 36 Watt headlight bulb?
  - A. 3 ohms
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  - C. 0.3 ohms
- Power of 36W =  $IV$ ,  $V = 12$  so  $I = 3$ . Then  $I = V/R$ .

# Clicker Question

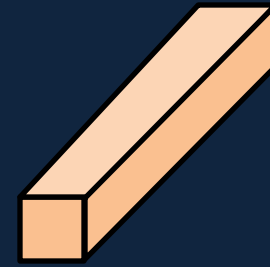
- Assume the 12V, 36 Watt headlight bulb has a tungsten filament. What is its approximate power output in the first instant it is connected, cold, to the 12V battery? ( $\rho_T \approx 15\rho_0$ ).
- A. 36W
- B. 2.4W
- C. 540W

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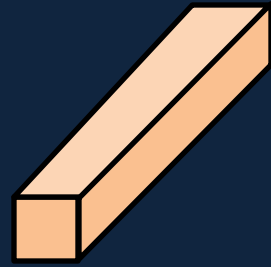
Power  $P = IV = V^2/R$ :  $R$  when initially cold is  $1/15$  of  $R$  at operating temperature of 3300K.

# Drift Speed



- Take a piece of copper wire, say 1mmx1mm cross section, 1m long carrying 5 amps.
- This is 1cc of Cu, about 10 gms, about  $10^{23}$  conduction electrons (one per atom), about **15,000C of electron charge**.
- Therefore, at 5 amps (C/sec) it takes **3000secs** for an electron to drift 1m.
- Bottom line: the drift velocity is of order  $10^{-4}$  m/sec. (it's linear in current, and depends on wire thickness for given current, obviously.)

# Drift Speed and Electron Speed



- Take a piece of copper wire, say 1mmx1mm cross section, 1m long carrying 5 amps: this wire has resistance  $R = \rho \ell / A \approx 0.02\Omega$  so from Ohm's law  $E \approx 0.1 \text{ V/m}$ .
- This field will accelerate the electrons,  $ma = eE$ , approximate accn =  $2 \times 10^{10} \text{ m/s}^2$ . This reaches drift velocity in about  $0.5 \times 10^{-14}$  seconds, that must be time between collisions.
- Electron speed (from quantum mechanics) is about  $2 \times 10^6 \text{ m/s}$ , so goes of order  $10^{-8} \text{ m}$  between collisions—past dozens of atoms.



# AC and DC

- **Batteries** provide **direct current, DC**: it always flows in the same direction.
- Almost all electric generators produce a voltage of **sine wave** form:

$$V = V_0 \sin 2\pi ft = V_0 \sin \omega t$$

- This drives an **alternating current, AC**,

$$I = \frac{V_0 \sin \omega t}{R} = I_0 \sin \omega t$$

and power

$$P = VI = I^2 R = I_0^2 R \sin^2 \omega t = \left( V_0^2 / R \right) \sin^2 \omega t$$

# AC Average Power and rms Values

- The AC power  $P = (V_0^2 / R) \sin^2 \omega t$  varies rapidly ( $\omega = 2\pi f$ ,  $f = 60$  Hz here), what is significant for most uses is the **average** power.
- The average value of  $\sin^2 \omega t$  is  $\frac{1}{2}$ .
- Define  $V_{\text{rms}}$  by  $V_{\text{rms}} = \sqrt{\overline{V^2}} = V_0 / \sqrt{2}$
- Then the average power  $\bar{P} = V_{\text{rms}}^2 / R$

average value of  $\sin^2 \omega t$  must equal average value of  $\cos^2 \omega t$ . and remember  $\sin^2 \omega t + \cos^2 \omega t = 1$

The **standard 120V** AC power is  $V_{\text{rms}} = 120\text{V}$ .

So the **maximum voltage  $V_0$**  on a 120V line is  $120 \times \sqrt{2} = 170\text{V}$ !

# Why Bother with AC?

- Because, as we'll discuss a little later, it's very easy to transform from high voltage to low voltage using transformers. →
- This means very high voltages can be used for longer distance transmission, low voltages for local use.



# Clicker Question

- The resistivity of aluminum is 58% higher than that of copper.
- A **copper** high voltage line has **diameter 1 cm**. If is replaced by an **aluminum** line of the same resistance, the aluminum line has **diameter**:
  - A. 1.58cm
  - B. 1.27cm
  - C. 0.8 cm
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Remember  $R = \rho L/A$ . The power lines have the same length, the aluminum therefore needs 58% more cross-section area  $A$ , from which diameter up by factor  $\sqrt{1.58}$ .

# High Voltage Power Lines ...

- Are made of **aluminum**—you need 58% more than copper by volume, but less than half the weight, *and* it's about 65% cheaper per kg.
- No contest.
- Some steel may be added for strength.

