Electric Currents and Resistance

Physics 2415 Lecture 10

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Today's Topics

- First we'll finish capacitors
- Then current electricity: frogs' legs, etc.
- The lithium ion battery
- Circuits and currents: Ohm's law
- Power usage: kWh, etc.

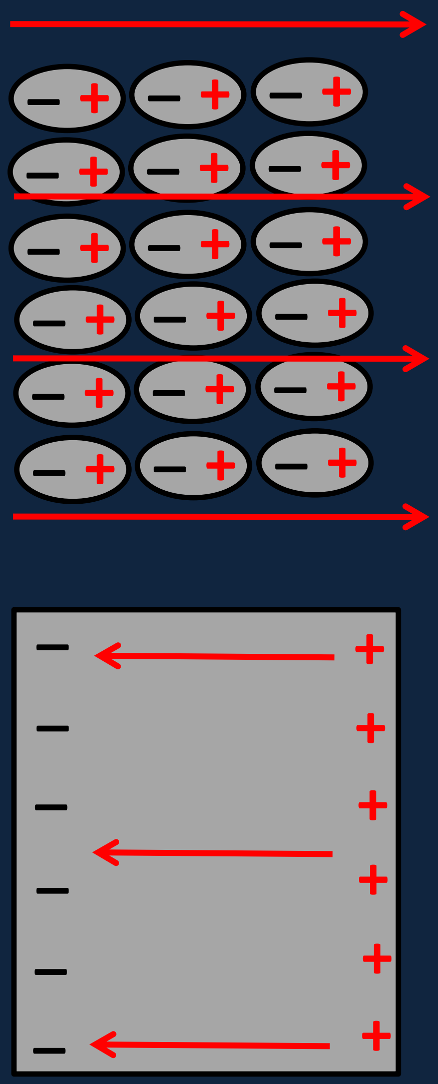
Energy Stored in a Capacitor

- The work needed to place charge in a capacitor is stored as electrostatic potential energy in the capacitor:

$$U = \frac{Q^2}{2C} = \tfrac{1}{2} C V^2 = \tfrac{1}{2} Q V$$

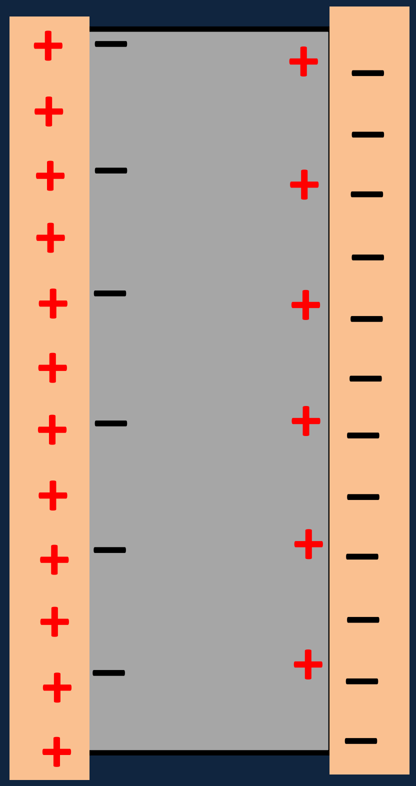
Dielectrics

- The "layers of surface excess charge" created by the polarization generate an electric field opposing the external field.
- However, unlike a conductor, this field cannot be strong enough to give zero field inside, because then the polarization would all go away.



Dielectric in a Capacitor

- If a dielectric material is placed between the parallel plates of a capacitor, the effect of the dielectric produced "surface layers of charge" is to partially cancel the charge on the plates as seen from inside the capacitor.
- Therefore, the dielectric will reduce the electric field strength, and therefore the voltage between the plates for given Q capacitor charge.



The Dielectric Constant K

- It is found experimentally that putting dielectric material between the plates of a capacitor reduces the magnitude of the electric field by a constant K that varies with the material used.
- This means that it takes more plate charge to give the same voltage: in other words, <u>the capacitance increases by a factor K</u>.

TABLE 24–1
Dielectric Constants (at 20°C)

Material	Dielectric constant K	Dielectric strength (V/m)
Vacuum	1.0000	
Air (1 atm)	1.0006	3×10^6
Paraffin	2.2	10×10^6
Polystyrene	2.6	24×10^6
Vinyl (plastic)	2–4	50×10^6
Paper	3.7	15×10^6
Quartz	4.3	8×10^6
Oil	4	12×10^6
Glass, Pyrex	5	14×10^6
Porcelain	6–8	5×10^6
Mica	7	150×10^6
Water (liquid)	80	
Strontium titanate	300	8×10^6

Energy Storage in a Dielectric

- Inserting dielectric between the plates of a capacitor increases the capacitance from C_0 to KC_0.
- This means that the energy stored at voltage V goes from $\frac{1}{2}C_0V^2$ to $\frac{1}{2}KC_0V^2$: yet inside the capacitor, the electric field has the same strength, V/d, as before. Where is the extra energy stored?
- In the dielectric: the stretched molecules store energy like little springs, so the total energy density of a field in a dielectric is

$$u = \frac{1}{2}K\varepsilon_0 E^2 = \frac{1}{2}\varepsilon E^2$$

Note: ε is called the permittivity of the material

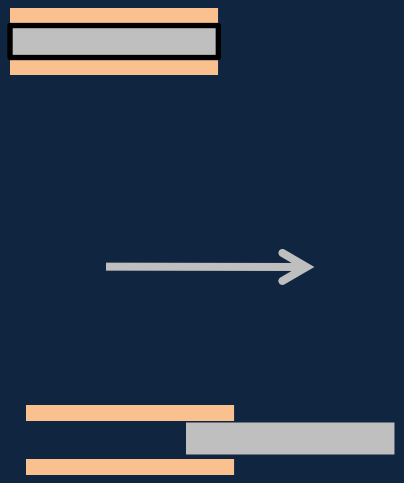
Clicker Question

- An isolated (no battery connection) parallel plate capacitor has charges $+Q$, $-Q$ on its plates, and dielectric ($K = 3$) between them.
- The dielectric is now removed, without disturbing the charge on the plates.
- The capacitor's energy has:

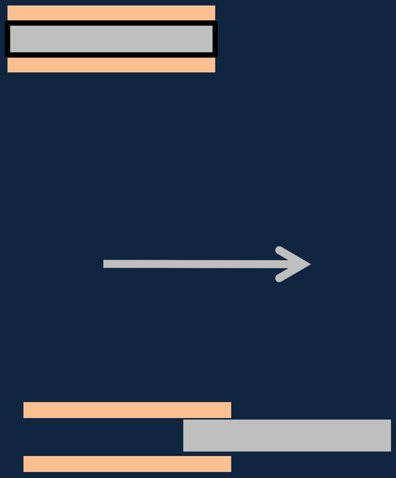
A. increased.

B. decreased.

C. stayed the same.



Clicker Answer



- An isolated (no battery connection) parallel plate capacitor has charges $+Q$, $-Q$ on its plates, and dielectric (K = 3) between them.
- The dielectric is now removed, without disturbing the charge on the plates.
- The capacitor's energy has:

A. Increased.

$U = Q^2/2C$, Q is constant, C decreases.

(The charge on the dielectric surface attracts that on the plates, so it takes work to separate them.)

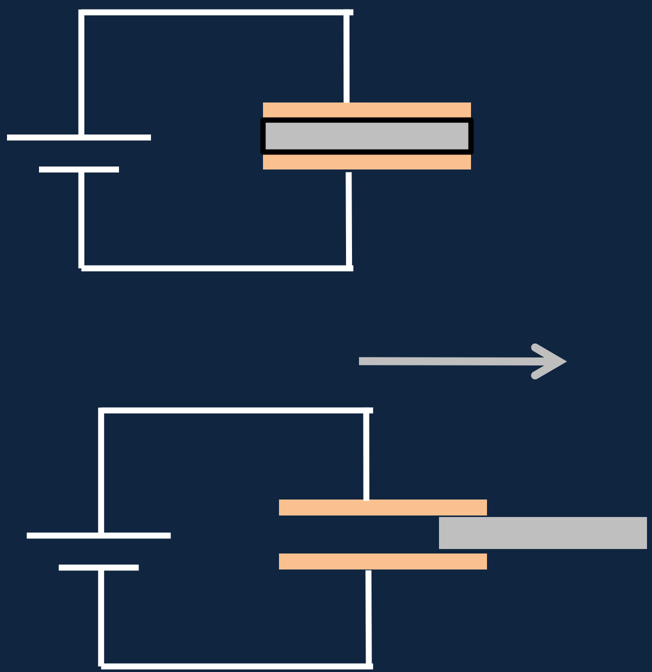
Clicker Question

- A parallel plate capacitor has its plates connected to a 100V battery, and dielectric (K = 3) between them.
- The dielectric is now removed, while keeping the 100V battery connection.
- The capacitor's energy has:

A. increased.

B. decreased.

C. stayed the same.

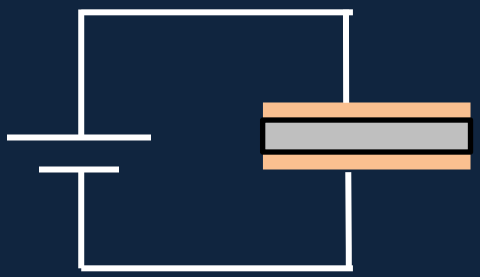


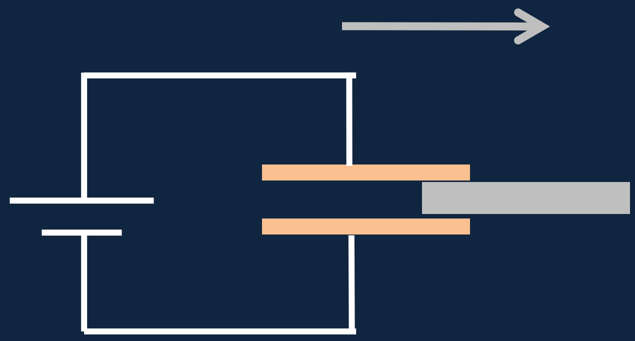
Clicker Answer

- A parallel plate capacitor has its plates connected to a 100V battery, and dielectric ($K = 3$) between them.
- The dielectric is now removed, while keeping the 100V battery connection.
- The capacitor's energy has:

B. Decreased.

- $U = ½CV^2$, V is constant, C decreases.





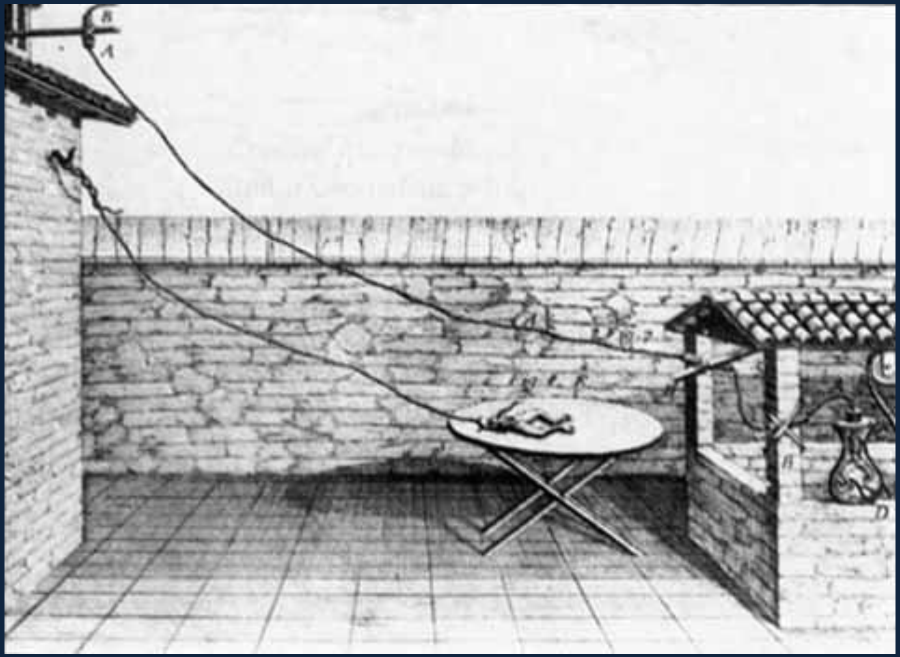
It still took work—but now you're charging the battery!

Capacitor Driven Bus

- Bus energy is stored in a capacitor (about 1kF).
- Recharges in two minutes at stops every two miles. (Those overhead wires are only at recharging station).
- Recharges much faster than batteries—but only 10% storage capacity/kg currently.

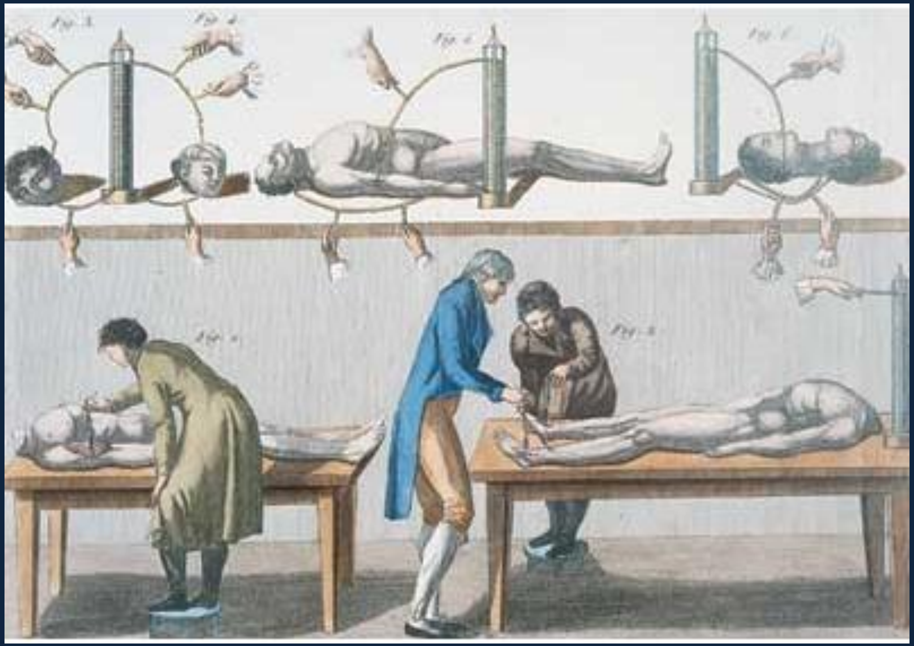
Electricity and Frog's Legs

- In1771, Luigi Galvani, at the University of Bologna, was dissecting frog's legs at a table that also had an electrostatic generator. He found by accident that the legs twitched in response to a charge, and were far more sensitive than the best electroscopes. He tried to detect atmospheric electricity.
- He found instead that electricity was generated by touching the legs with dissimilar metals.



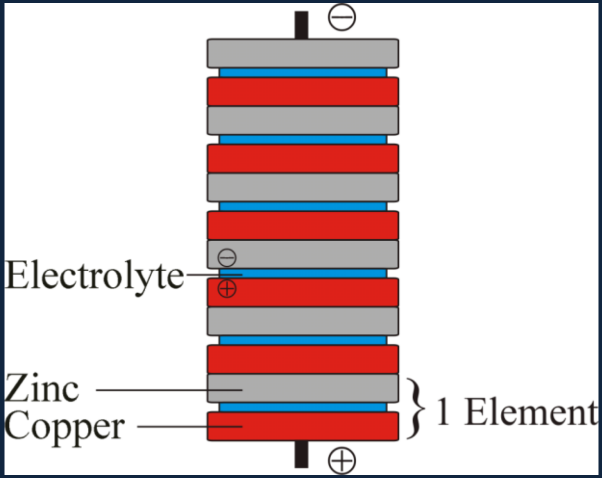
Reviving Dead Criminals?

- Galvani's nephew Giovanni Aldini, a showman, electrified corpses just after decapitation at a prison in London, with various muscular reactions.
- This was the inspiration behind Frankenstein.



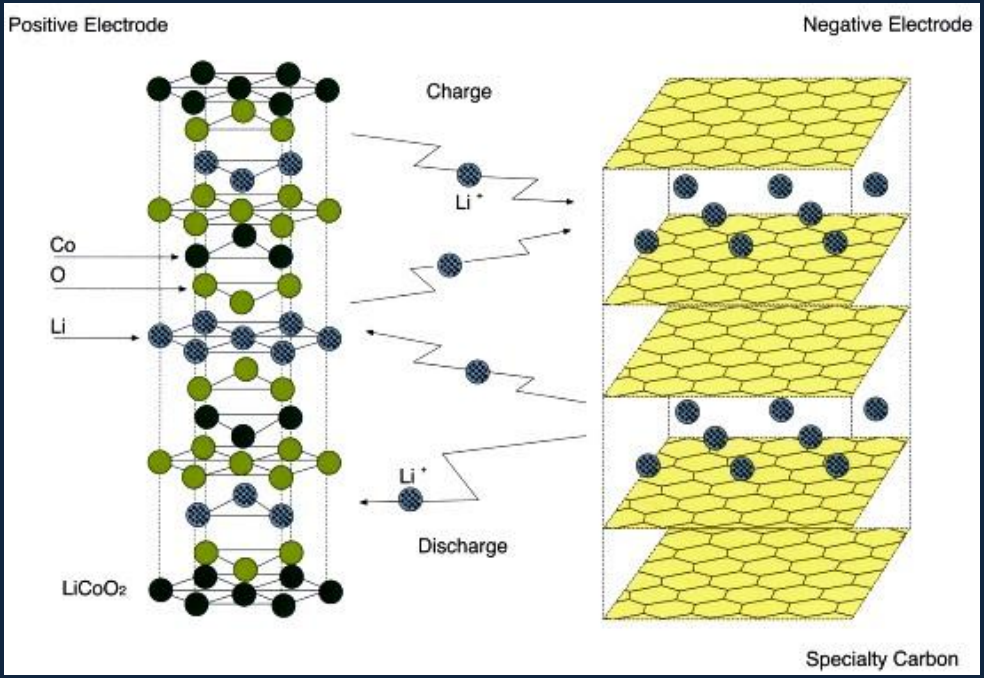
Volta's Pile

- Galvani's colleague Volta was the first to realize that using different metals to touch the frog's leg was crucial to producing electricity, and in fact the leg could be replaced with cardboard soaked in brine.
- He built a pile of such metal pairs—the first such battery—with dubious medical applications…



A Modern Battery: Lithium Ion

- Lithium ions Li^+ are very tiny: remember H, He, Li, …they are He atoms with an extra nuclear charge. They can fit between atomic layers in graphite, to which they bond, but bond more strongly in $LiCoO_2$. Charging is by attracting them from the $LiCoO_2$ into the graphite by pumping in electrons.

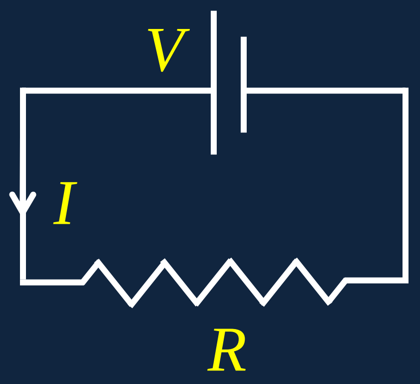


Batteries, Circuits, Currents

- The two terminals of a battery, called electrodes, are immersed in an electrolyte. Positive ions are formed at one electrode by atoms depositing electrons.
- For suitably chosen materials, energy is generated by these electrons flowing round an outside wire to take part in a chemical reaction (or just rejoin the ions) at the other electrode.
- The "outside wire" is the circuit. Flow is measured in coulombs per sec, called Amperes.

Ohm's Law

- Ohm found experimentally in 1825 that for a given piece of wire, the current, labeled I, was directly proportional to the applied voltage (number of battery cells) V, and wrote it as $I = V/R$, where V is in volts, I in amps.
- R is called the resistance of the wire, and is measured in ohms: one volt sends one amp through one ohm.



These are the standard symbols for a battery and a resistance: remember the standard "current" is really electrons flowing the other way!

Electric and Water Currents Compared

- It’s sometimes useful to think of electric current down a wire as resembling water flowing down a pipe.
- Pressure difference between two ends of a water pipe corresponds to voltage difference between the ends of a wire.
- Flow rate is determined by pressure gradient: a water pipe twice as long drops twice the pressure during flow, in electrical terms, a wire twice as long has twice the resistance.

Resistance and Cross-Section Area

- Suppose we take two identical wires, having the same area of cross section A, and twist them together to make one wire.
- When this is done, it's found (not surprising) that the combination delivers twice the current of a single wire for the same voltage.
- But effectively we've doubled the cross section area: so R is proportional to $1/A$.

Resistance and Resistivity

- To summarize: for a given material (say, copper) the resistance of a piece of uniform wire is proportional to its length ℓ and inversely proportional to its cross-sectional area A.
- This is written: $$R = \rho \frac{\ell}{A}$$

  where ρ is the resistivity.
- For copper, $\rho = 1.68 \times 10^{-8}\ \Omega \cdot \text{m}.$

Electric Power

- Remember voltage is a measure of potential energy of electric charge, and if one coulomb drops through a potential difference of one volt it loses one joule of potential energy.
- So a current of I amps flowing through a wire with V volts potential difference between the ends is losing IV joules per sec.
- This energy appears as heat in the wire: the electric field accelerates the electrons, which then bump into impurities and defects in the wire, and are slowed down to begin accelerating again, like a sloping pinball machine.

Power and Energy Usage

- Using Ohm's law, we can write the power use of a resistive heater (or equivalent device, such as a bulb) in different ways:

$$P = IV = I^2 R = V^2 / R$$

- The unit is watts, meaning joules per second.
- Electric meters measure total energy usage: adding up how much power is drawn for how long, the standard unit is the kilowatt hour:
- 1 kWh = 1,000x3,600J = 3.6MJ