Capacitors

Physics 2415 Lecture 8

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Today's Topics

- Dipole Potential, Lightning Conductors
- Storing Charge on a Spherical Conductor
- Parallel Plate Capacitors
- Cylindrical Capacitors
- Capacitors in Series and Parallel

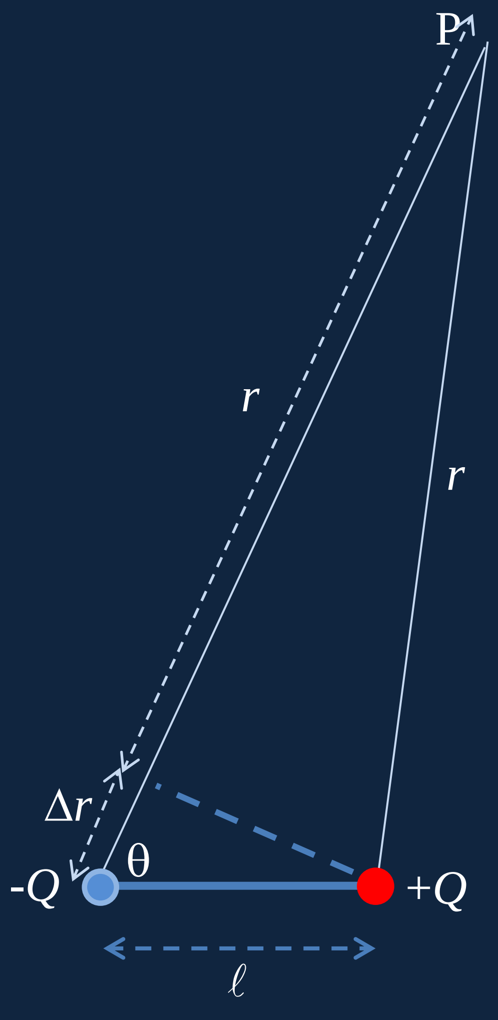
Dipole Potential Far Away

- At distances $r \gg \ell$, the charge separation distance, the dipole potential at a point P has a simple form:

$$V(\mathrm{P}) = \frac{kQ}{r} - \frac{kQ}{r+\Delta r} = \frac{kQ\Delta r}{r(r+\Delta r)}$$

$$\cong \frac{kQ\ell\cos\theta}{r^2} \cong \frac{kp\cos\theta}{r^2}$$

- Recall the dipole moment $p = Q\ell$, and we've approximated for $r \gg \Delta r$.

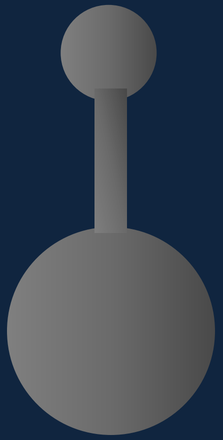


Connected Spherical Conductors

- Two spherical conductors are connected by a conducting rod, then charged—all will be at the same potential.
- Where is the electric field strongest?

A. At the surface of the small sphere.

- Take the big sphere to have radius R_1 and charge Q_1, the small R_2 and Q_2.
- Equal potentials means $Q_1/R_1 = Q_2/R_2$.
- Since $R_1 > R_2$, field $kQ_1/R_1{}^2 < kQ_2/R_2{}^2$.
- This means the surface charge density is greater on the smaller sphere!



Electric Breakdown of Air

- Air contains free electrons, from molecules ionized by cosmic rays or natural radioactivity.
- In a strong electric field, these electrons will accelerate, then collide with molecules. If they pick up enough KE between collisions to ionize a molecule, there is a "chain reaction" with rapid current buildup.
- This happens for E about 3×10^6 V/m.

Photograph of lightning striking the Eiffel Tower, June 3, 1902, taken by M.G. Loppé.

Voltage Needed for Electric Breakdown

- Suppose we have a sphere of radius 10cm, 0.1m.
- If the field at its surface is just sufficient for breakdown,

$$3 \times 10^6 = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^2}$$

- The voltage

$$V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R} = 3 \times 10^6 R = 300,000V$$

- For a sphere of radius 1mm, 3,000V is enough—there is discharge before much charge builds up.
- This is why lightning conductors are pointed!

Charged Sphere Potential and Field

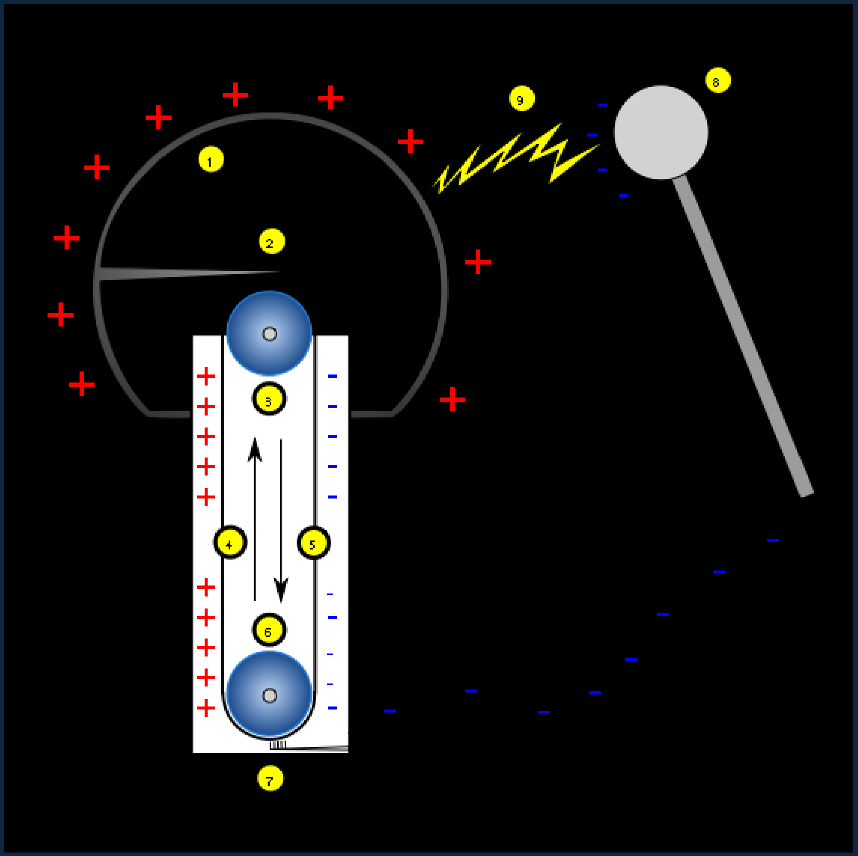
- For a spherical conductor of radius R with total charge Q uniformly distributed over its surface, we know that

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q\hat{r}}{r^2} \quad \text{and} \quad V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}.$$

- The field at the surface is related to the surface charge density σ by $E = \sigma/\varepsilon_0$.
- Note this checks with $Q = 4\pi R^2\sigma$.

Storing Charge on a Sphere

- In the van der Graaff machine, electric charge is carried up to a sphere on an insulating belt, the belt drive overcoming the electrostatic repulsion.
- As charge builds up, so does the voltage—to the point where breakdown of air or part of the insulator occurs.



Capacitance of a Sphere

- Recall that the potential (voltage) at the surface of the sphere is

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

for a sphere of radius R.

- Notice that the stored charge Q is directly proportional to the voltage V, and the ratio of the two is called the capacitance:

$$C = Q/V$$

Unit Capacitance

- Writing $C = Q/V$ *defines* the unit capacitance: adding one coulomb of charge must raise the potential of a capacitance with $C = 1$ by one volt.
- This unit capacitance is called one farad, 1F, in honor of Michael Faraday, the first person to understand electric and magnetic fields.
- For the spherical capacitor,

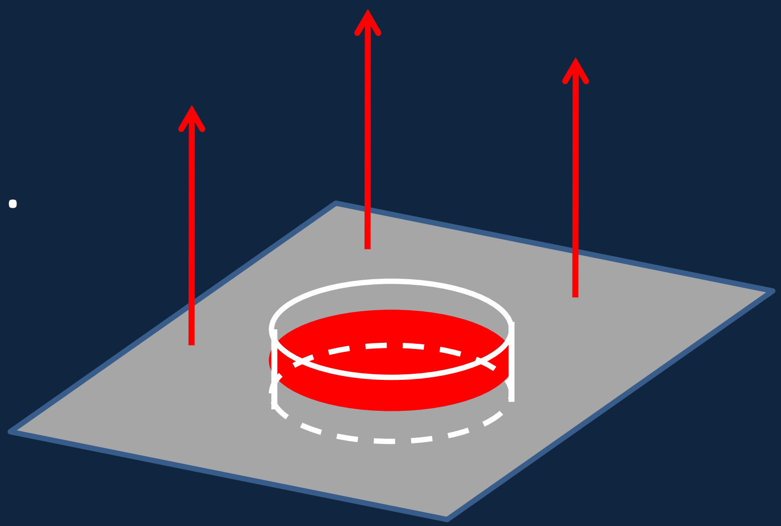
$$Q/V = 4\pi\varepsilon_0 R = R/\left(9\times10^9\right)$$

so a 1F sphere capacitor is bigger than the Sun!

Reminder

Charge on Surface of a Conductor

- For a flat conducting surface, the electric field is perpendicularly outward, or a current would arise.
- We have a sheet of charge on the surface, so we take the same Gaussian pillbox as for the sheet of charge, but this time there is no electric field pointing downwards into the conductor.
- Therefore Gauss' Law gives

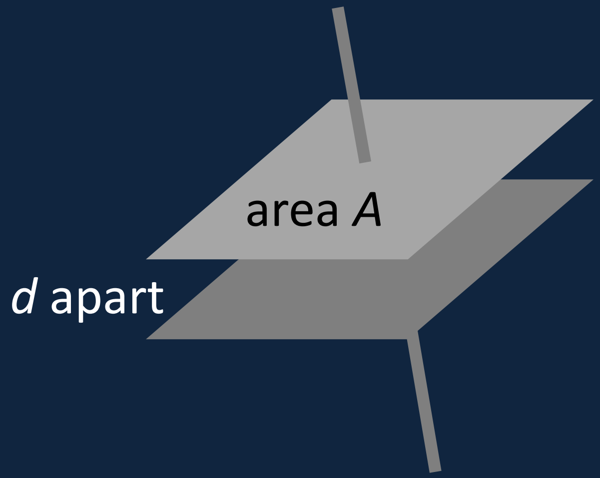


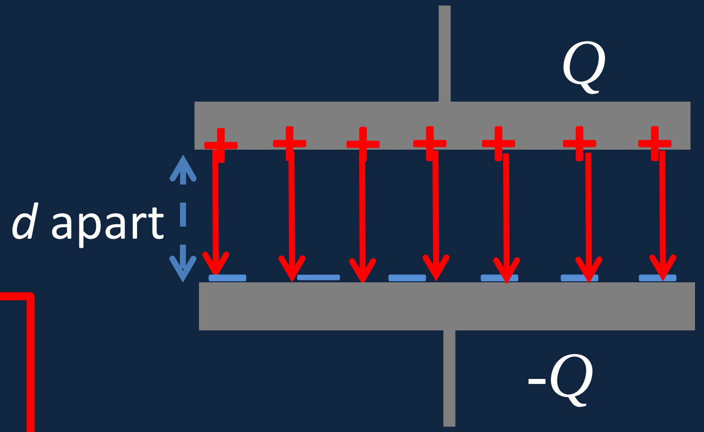
$$AE = \frac{\sigma A}{\varepsilon_0}, \text{ so } \boxed{E = \frac{\sigma}{\varepsilon_0}}$$

Parallel Plate Capacitor

- Almost all capacitors are parallel plate capacitors: two conducting plates each of area A a constant distance d apart.
- For total charge Q on the top plate and $-Q$ on the bottom, taking $d << \sqrt{A}$,
- $E = \sigma/\varepsilon_0 = Q/A\varepsilon_0$ and $V = Ed$, so

$$V = \frac{Qd}{A\varepsilon_0} = \frac{Q}{C} \text{ where } C = \varepsilon_0 \frac{A}{d}$$



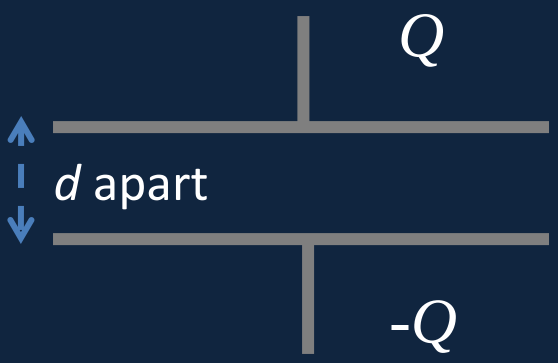


*Charge will settle on **inside** surfaces*

Clicker Question

- The two capacitors on the right have the same size plates, and hold the same charge Q.
- For which capacitor is the voltage V between the plates greater?

A:

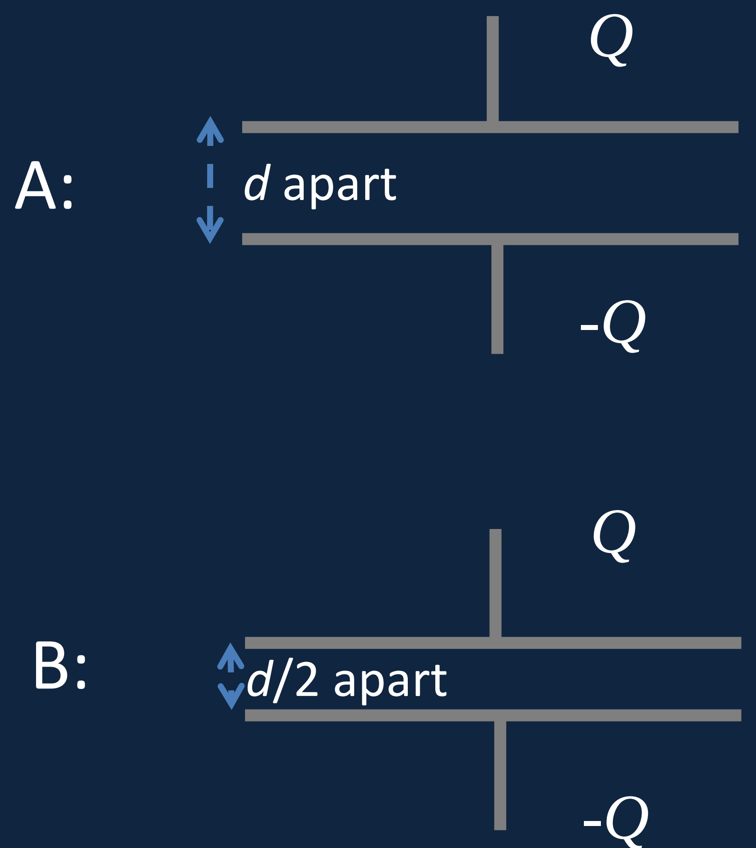


B:

C: they have the same V

Clicker Answer

- The two capacitors on the right have the same size plates, and hold the same charge Q.
- The same charge density means the same strength electric field between the plates: so to move a charge between plates for A takes twice the work—twice the voltage.



C: they have the same V

Clicker Question

- If the voltage applied to a parallel plate capacitor is doubled, what happens to the capacitance C?

A. It's doubled

B. It's halved

C. It doesn't change

Clicker Answer

- If the voltage applied to a parallel plate capacitor is doubled, what happens to the capacitance C?

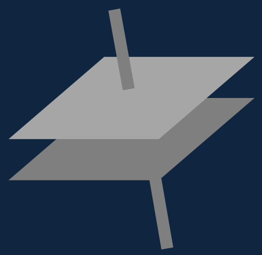
It doesn't change: capacitance depends on plate area and plate separation, NOT on the charge stored.

Don't use formulas before thinking, however briefly!

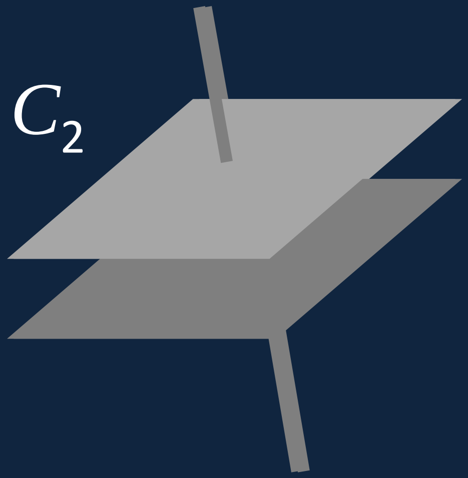
Clicker Question

- Capacitor C_2 is a scale model of capacitor C_1, with all linear dimensions up by a factor of 2.
- What is the ratio of capacitances?

C_1



C_2

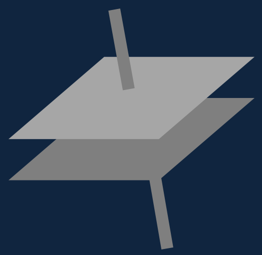


A. C_2 is 8 times C_1
B. C_2 is 4 times C_1
C. C_2 is twice C_1

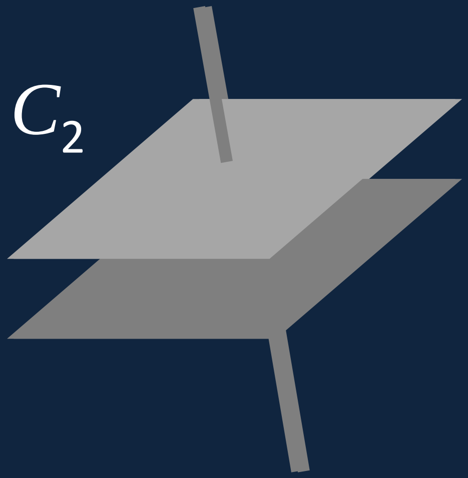
Clicker Answer

- Capacitor C_2 is a scale model of capacitor C_1, with all linear dimensions up by a factor of 2.
- What is the ratio of capacitances?

C_1



C_2



C_2 is twice C_1:

Area is up by a factor of 4, but doubling of separation distance halves capacitance.

Clicker Question

- A parallel plate capacitor, consisting simply of two metal plates held parallel, is charged with a battery to a voltage V, then the battery is disconnected.
- The plates are now physically pulled further apart : what happens to the voltage?

A. It stays the same

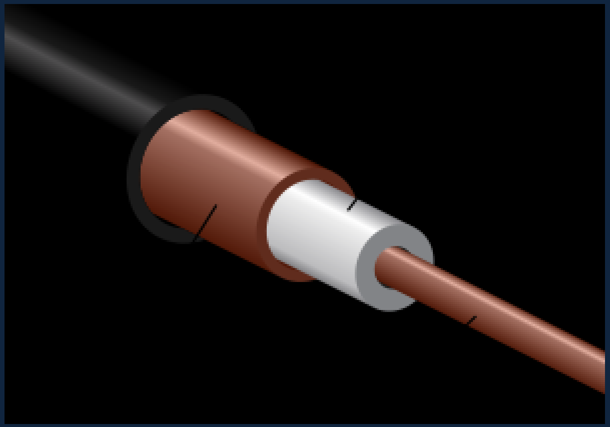
B. It decreases

C. It increases

Clicker Answer

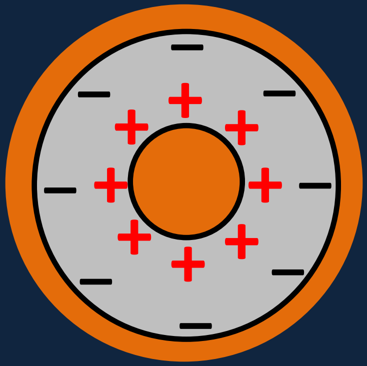
- A parallel plate capacitor, consisting simply of two metal plates held parallel, is charged with a battery to a voltage V, then the battery is disconnected.
- The plates are now physically pulled further apart : what happens to the voltage?
- It increases: the charge on the plates cannot change, so the electric field stays the same, and voltage is field strength x distance.

Cylindrical Capacitor



- A coaxial cable is a cylindrical capacitor.
- For charge density λ C/m on the inside wire (and so - λ on the inside of the outer cylinder) the radial field $E = \lambda/2\pi\varepsilon_0 r$ and

$$V = \int_{R_1}^{R_2} E(r)\,dr = \frac{\lambda}{2\pi\varepsilon_0}\int_{R_1}^{R_2}\frac{dr}{r} = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{R_2}{R_1}$$

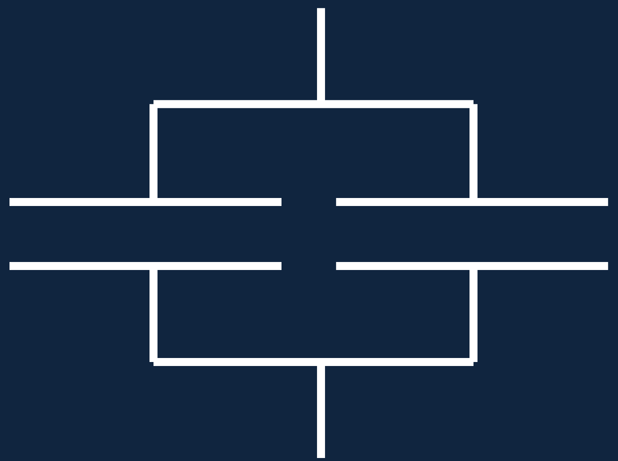


- so

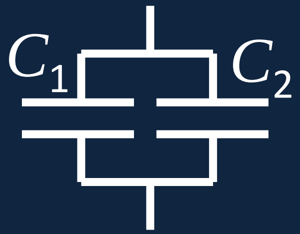
$$C = \frac{Q}{V} = \frac{2\pi\varepsilon_0 \ell}{\ln(R_2/R_1)} \text{ for length } \ell$$

Capacitors in Parallel

- Let's look first at hooking up two identical parallel plate capacitors in parallel: that means the wires from the two top plates are joined, similarly at the bottom, so effectively they become one capacitor.
- What is its capacitance? From the picture, the combined capacitor has twice the area of plates, the same distance apart.
- We see that $C = C_1 + C_2$



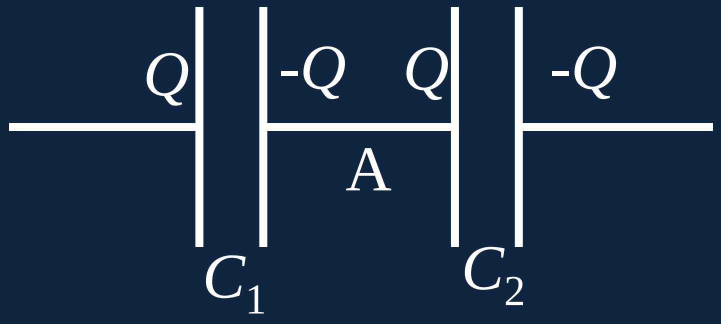
Capacitors in Parallel



- If two capacitors C_1, C_2 are wired together as shown they have the same voltage V between plates.
- Hence they hold charges $Q_1 = C_1V$, $Q_2 = C_2V$, for total charge $Q = Q_1 + Q_2 = (C_1 + C_2)V = CV$.
- So capacitors in parallel just add:

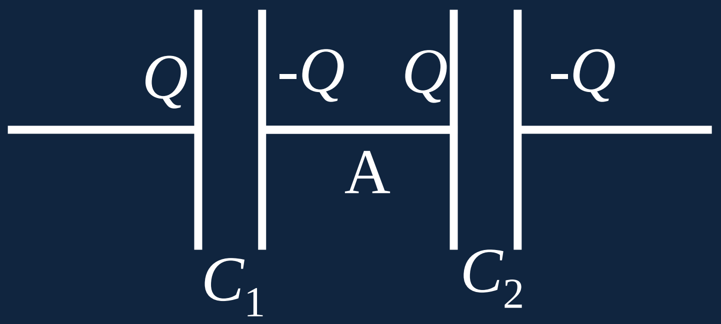
$$C = C_1 + C_2 + C_3 + \ldots$$

Capacitors in Series



- Regarding the above as a single capacitor, the important thing to realize is that in adding charge via the outside end wires, no charge is added to the central section labeled A: it's isolated by the gaps between the plates.
- Charge Q on the outside plate of C_1 will attract $-Q$ to the other plate, this has to come from C_2, as shown.
- <u>Series capacitors all hold the same charge.</u>

Capacitors in Series



- Series capacitors all hold the same charge.
- The voltage drop V_1 across C_1 is $V_1 = Q/C_1$.
- The voltage drop across C_2 is $V_2 = Q/C_2$.
- Denoting the total capacitance of the two taken together as C, then the total voltage drop is $V = Q/C$.
- But $V = V_1 + V_2$, so $Q/C = Q/C_1 + Q/C_2$,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$