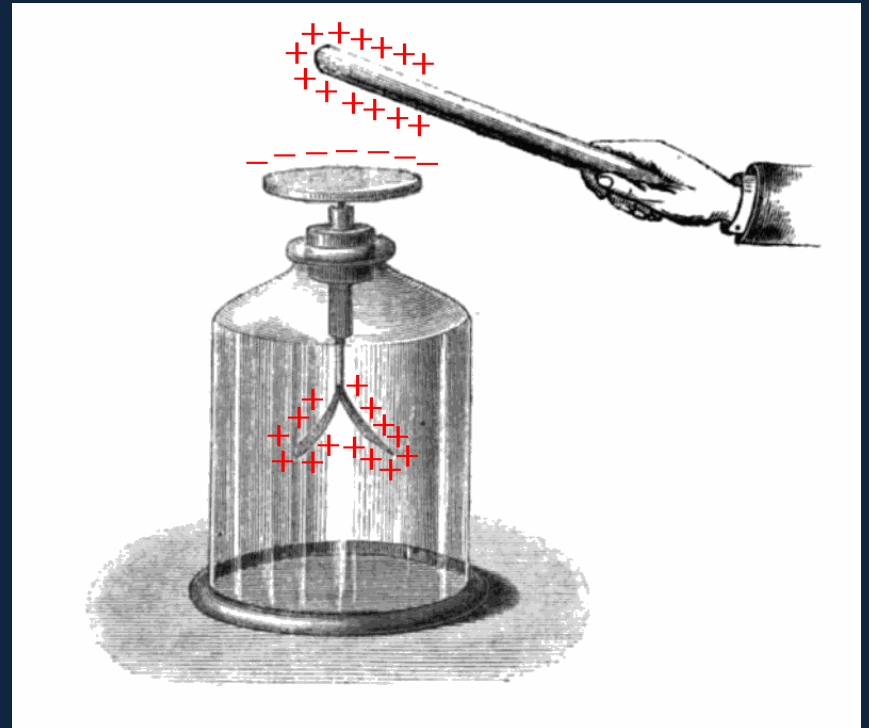# Coulomb's Law and the Electric Field

Physics 2415 Lecture 2

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The Electroscope

- Charge detector invented by an English clergyman in 1787. Two very thin strips of gold leaf hang side by side from a conducting rod.

- If a **+** charge is brought near, electrons move up the rod, leaving the two strips positively charged, so they repel each other.

Charging the Electroscope…

- By conduction: touch the top conductor with a positively charged object—this will leave it positively charged (electron deficient).

- By induction: while holding a positively charged object near, but not in contact, with the top, you touch the electroscope: negative charge will flow from the ground, through you, to the electroscope.

Coulomb's Law

- Coulomb measured the electrical force between charged spheres with apparatus exactly like Cavendish's measurement of *G*: two spheres, like a dumbbell, suspended by a thin wire. One sphere was charged, another charged sphere was brought close, the angle of twist of the wire measured the force.
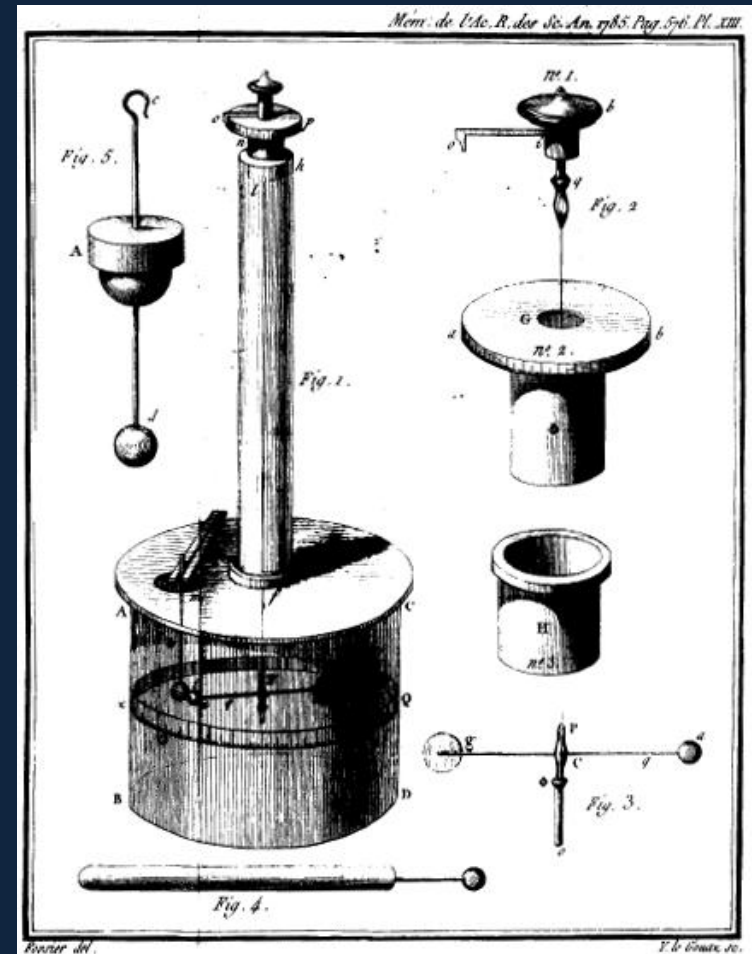
Coulomb's Law

- Coulomb discovered an inverse square law, like gravitation, except that of course like charges *repelled* each other. The force acted along the line of centers, with magnitude proportional to the magnitudes of both charges:

$$\vec{F}_{12}$$ $$+$$ $$Q_1$$

$$\vec{F}_{21}$$ $$+$$ $$Q_2$$

$$+$$ $$-$$
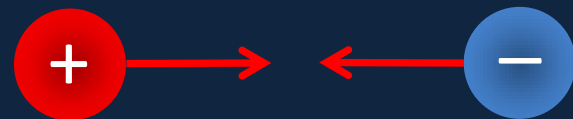
$$F = k\frac{Q_1 Q_2}{r^2}$$

Unit of Charge

- We can't make further progress until we define a unit of charge.

- The SI unit is the Coulomb. Its definition is <u>not</u> from electrostatics, but the SI unit current in a wire, one amp, is one coulomb per second passing a fixed point, and one amp is the current that exerts on an identical parallel current one meter away a magnetic force of one Newton per meter of wire. We'll do all this later—just letting you know why we have this very large unit.
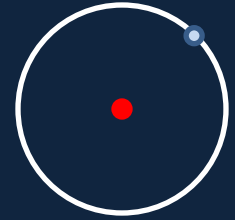
Coulomb's Law with Numbers

$$F = k\frac{Q_1 Q_2}{r^2}$$

$\vec{F}_{12}$ $\vec{F}_{21}$

$+$ $+$

Q_1 Q_2

$+$ $-$

- Experimentally, with *r* in meters and *F* in Newtons, it is found that *k* = 9x10^9.

- This means that two charges each one millicoulomb (10^{-3} C), one meter apart, repel with a force of 9,000N, about <u>one ton</u> weight!
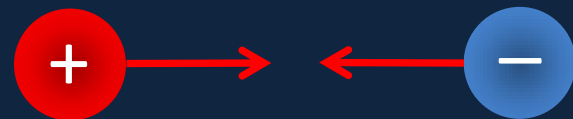
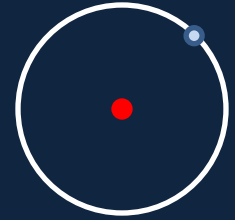<u>Note</u>: a common notation is $k = \dfrac{1}{4\pi\varepsilon_0}$

Atomic Electrostatics

- The simplest (Bohr) model of the hydrogen atom has an electron circling a proton at a distance of about 0.5×10^{-10} m.

- The electron charge has been determined experimentally to be about -1.6×10^{-19} C.

- This means the electrostatic force holding the electron in orbit

$$F = k \frac{Q_1 Q_2}{r^2}$$

is about 10^{-7} N.

Atomic Dynamics

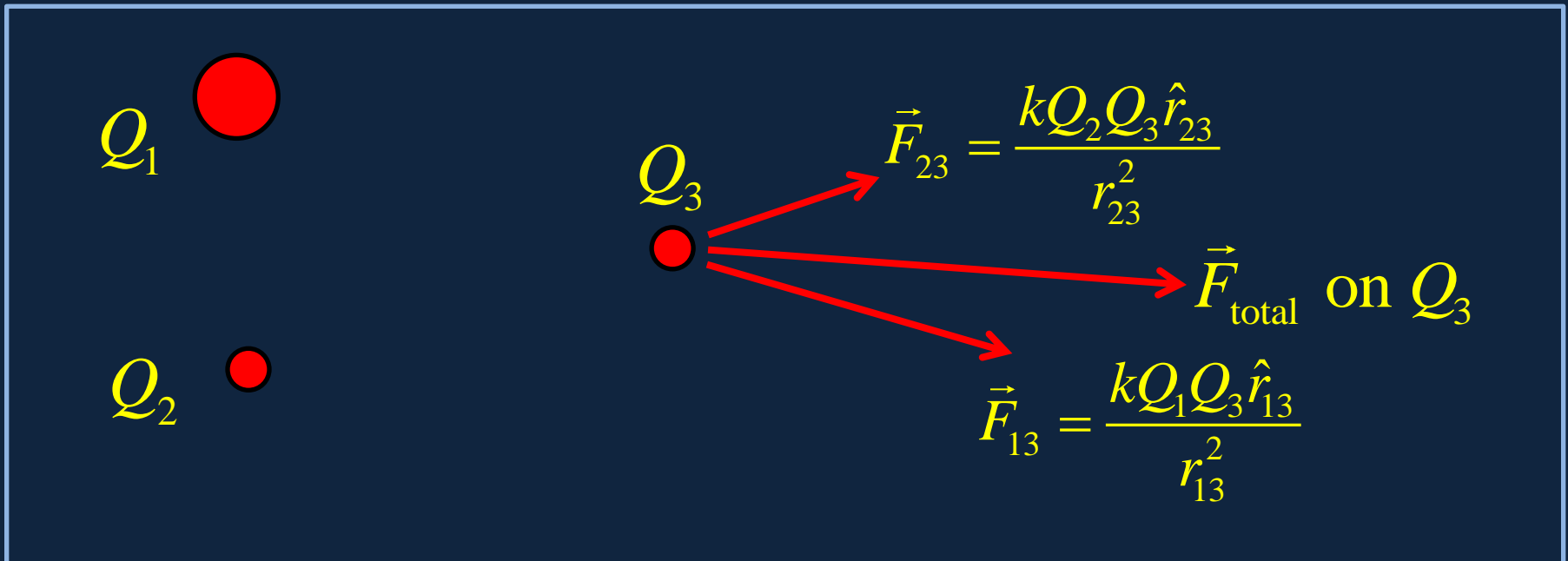- The electrostatic force holding the electron in orbit

$$F = k\frac{Q_1 Q_2}{r^2}$$

  is about 10^{-7}N.

- The electron has mass about 10^{-30} kg, so its acceleration is about 10^{23} m/s^2.

- This is v^2/r, from which v is about $2\text{x}10^6$ m/s, around 1% of the speed of light.

Superposition

- The total electric force on a charge Q_3 from two charges Q_1, Q_2 is the <u>vector sum</u> of the forces from the charges found separately.

$$Q_1$$

$$Q_3$$

$$\vec{F}_{23} = \frac{kQ_2Q_3\hat{r}_{23}}{r_{23}^2}$$

$$\vec{F}_{\text{total}} \text{ on } Q_3$$

$$Q_2$$

$$\vec{F}_{13} = \frac{kQ_1Q_3\hat{r}_{13}}{r_{13}^2}$$

Sounds trivial—but superposition *isn't true* for nuclear forces!

The Electric Field

- The electric field $\vec{E}(\vec{r})$ at a point $\vec{r}$ is <u>defined</u> by stipulating that the electric force $\vec{F}$ on a tiny test charge q at $\vec{r}$ is given by $\vec{F} = q\vec{E}$.

- Strictly speaking, the test charge should be *vanishingly small*: the problem is that if the electric field arises in part from charges on conductors, introducing the test charge could cause them to move around and thus change the field you're trying to measure.

Field from Two Equal Charges

- Two charges Q are placed on the y-axis, equal distances d from the origin up and down. What is the electric field at a point P on the x-axis, and where is its maximum value?
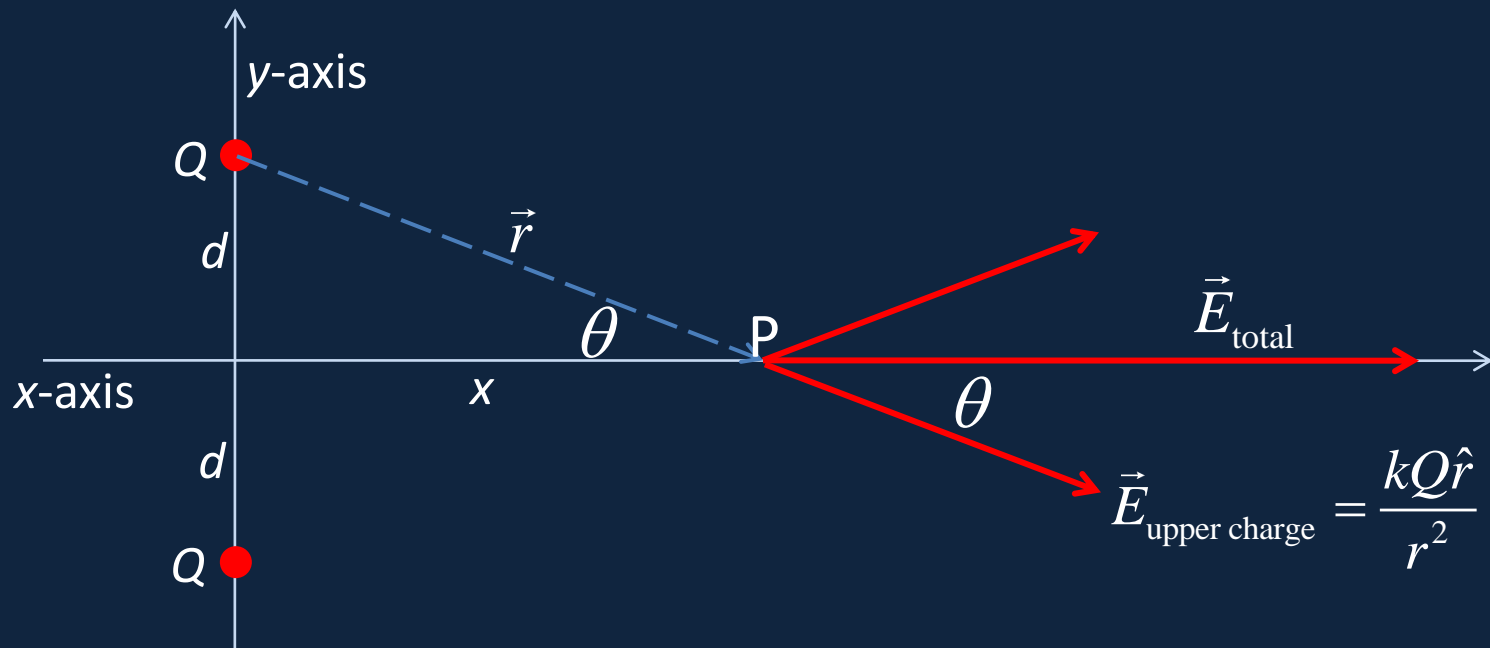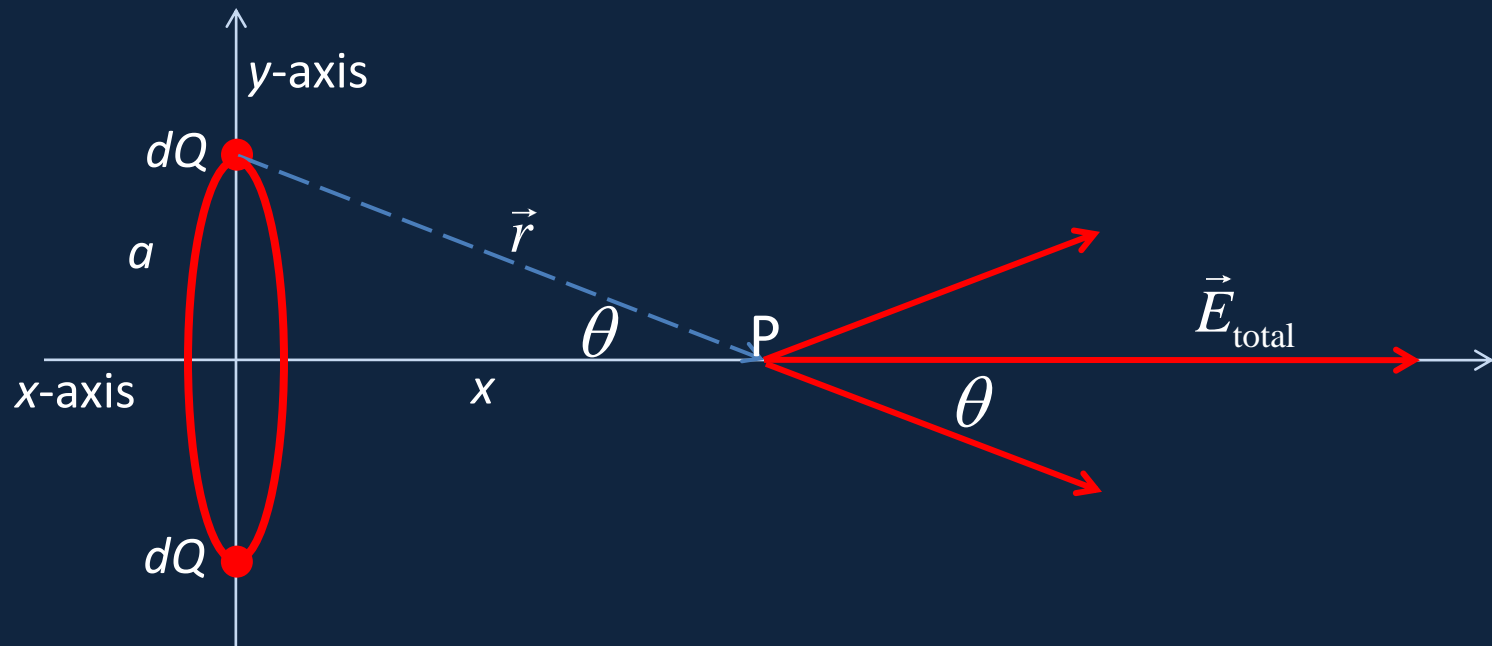

- Anywhere on the axis, the field is along the axis, and has value

$$E = \frac{2kQ}{r^2} \cos\theta = \frac{2kQx}{r^3} = \frac{2kQx}{\left(x^2 + d^2\right)^{3/2}}.$$

Field on the Axis of a Uniform Ring of Charge

- Imagine the ring, radius a, total charge Q, to be made up of pairs dQ of oppositely placed charges:
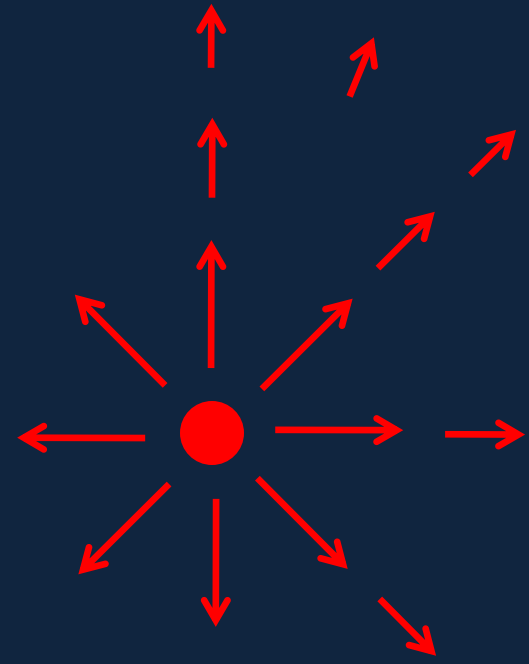

- From the previous slide, adding contributions from all pairs,

$$E = \frac{kQ}{r^2}\cos\theta = \frac{kQx}{r^3} = \frac{kQx}{\left(x^2 + a^2\right)^{3/2}}.$$
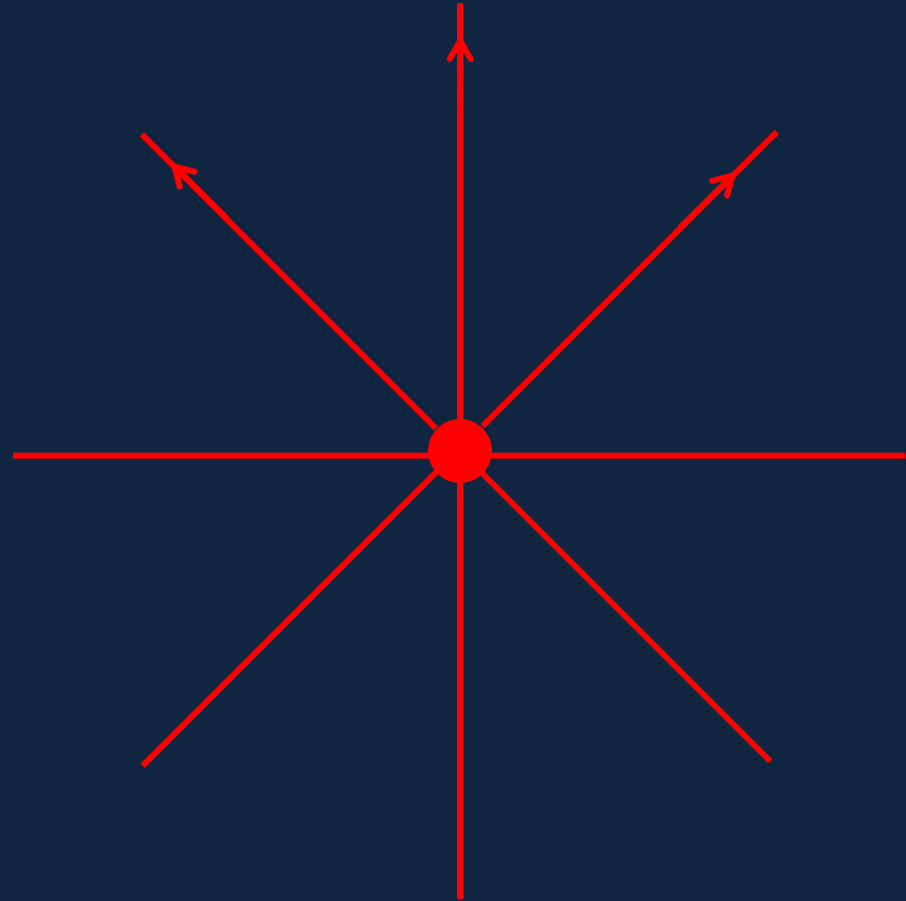
Visualizing the Electric Field

- For a single point charge, we can easily draw vectors at various points indicating the strength of the field there:

Visualizing the Electric Field

- A standard approach is to draw lines of force: lines that at every point indicate the field direction there.

- These lines do not immediately give the field strength, but their density can give a *qualitative* indication of where the field is stronger, provided they are continuous.

Field from a Uniform *Line* of Charge

- What's the electric field at a point P distance R from a very long line of charge, say λ C/m?

- Take the wire along the *z*-axis in 3D Cartesian coordinates, we'll find the field at a point P, distance R from the wire, in the (*x,y*) plane.

- The strategy is to find the field $d\vec{E}$ from a bit *dz* of the wire, then do an integral over the whole wire.

z-axis

dz

$R\sec\theta$

$R\tan\theta$

θ

P

O

R

$d\vec{E}$ from charge
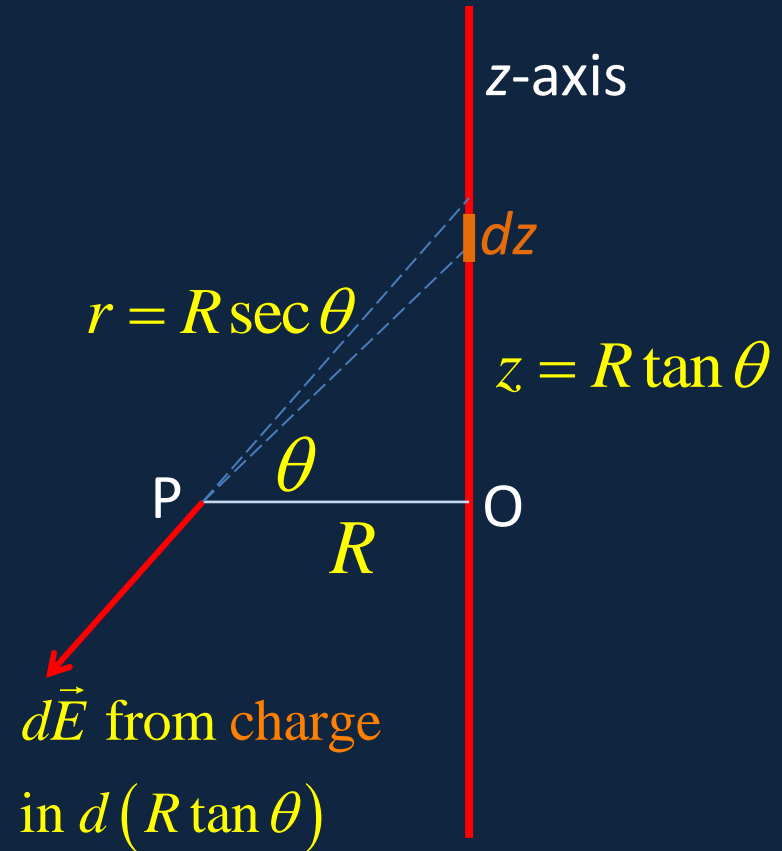
in $d\left(R\tan\theta\right)$
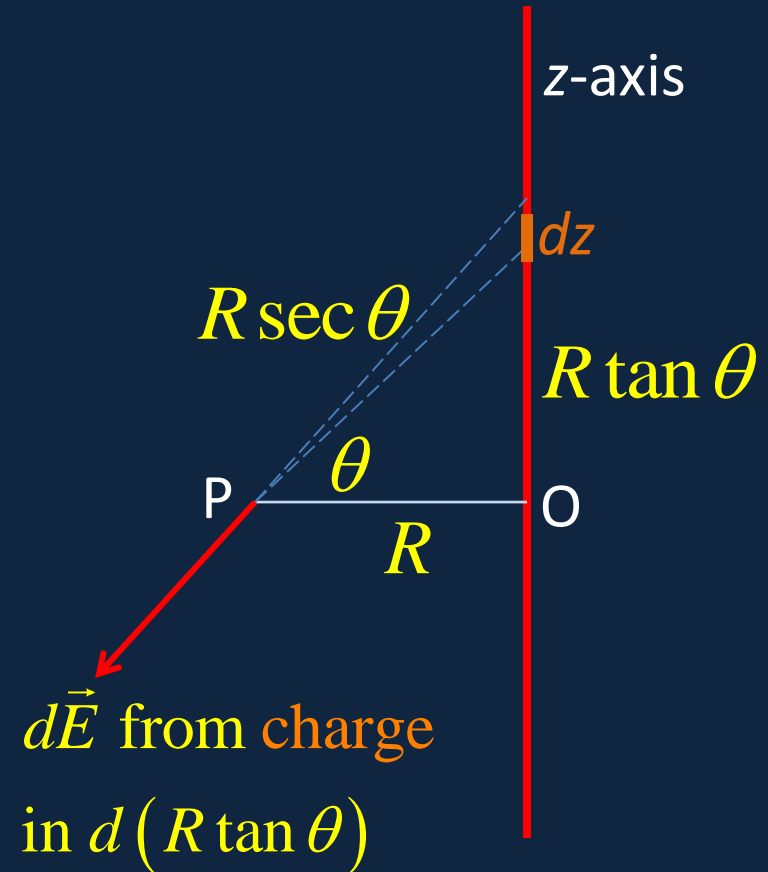
Field from a Uniform *Line* of Charge

- The strategy is to find the field $d\vec{E}$ from a bit *dz* of the wire, then do an integral over the whole wire.

$$\left| d\vec{E} \right| = \frac{kdq}{r^2} = \frac{k\lambda dz}{r^2} = \frac{k\lambda d\left(R\tan\theta \right)}{R^2\sec^2\theta} = \frac{k\lambda d\theta}{R}$$

- For an infinite wire, the net field must be directly away from the wire, so multiply by $\cos\theta$ and integrate over all *z* :

$$E = \frac{k\lambda}{R} \int\limits_{-\pi/2}^{\pi/2} \cos\theta\, d\theta = \frac{2k\lambda}{R}$$

z-axis

dz

$r = R\sec\theta$

$z = R\tan\theta$

θ

P

O

R

$d\vec{E}$ from charge

in $d\left(R\tan\theta \right)$

Electric Field from a *Line* of Charge: Top View

- It looks just like the field from a point charge: but isn't!

- Remember that for a point charge Q, the magnitude of field at distance R is kQ/R^2, for a line charge with density λ, the field strength is $k\lambda/2R$: so the "density of lines" in these 2D plots can't relate directly to field strength for both cases.

- (Actually, if we could draw the lines in 3D, the density *would* relate directly to field strength.)
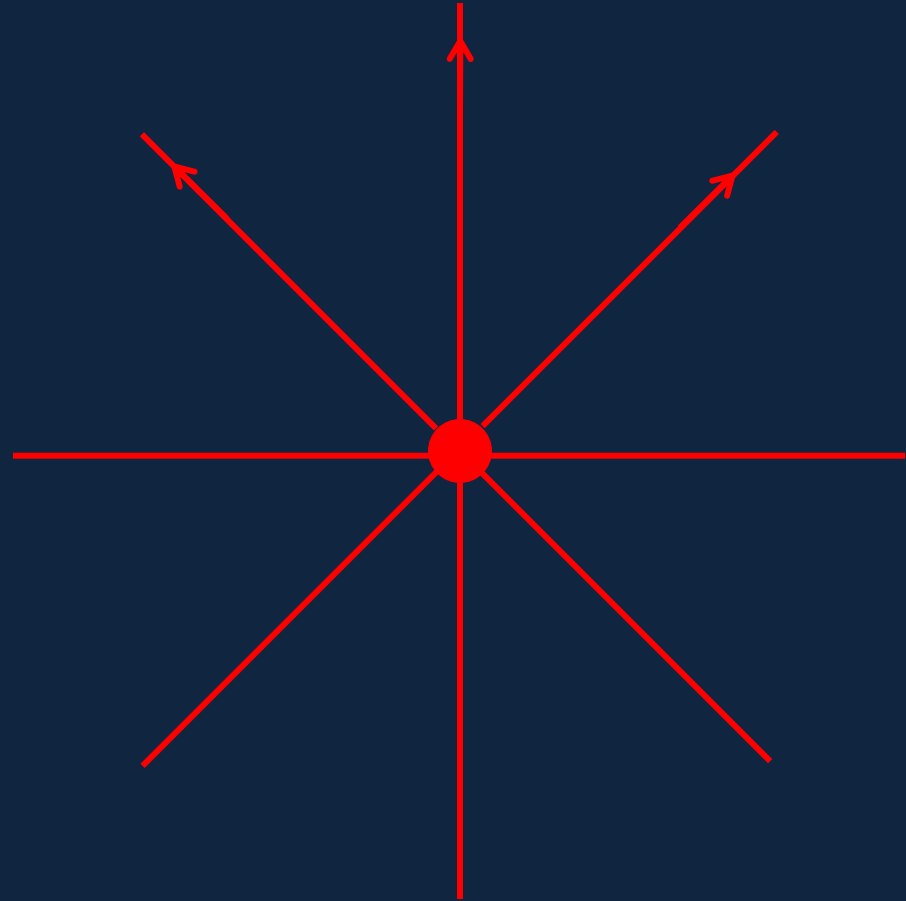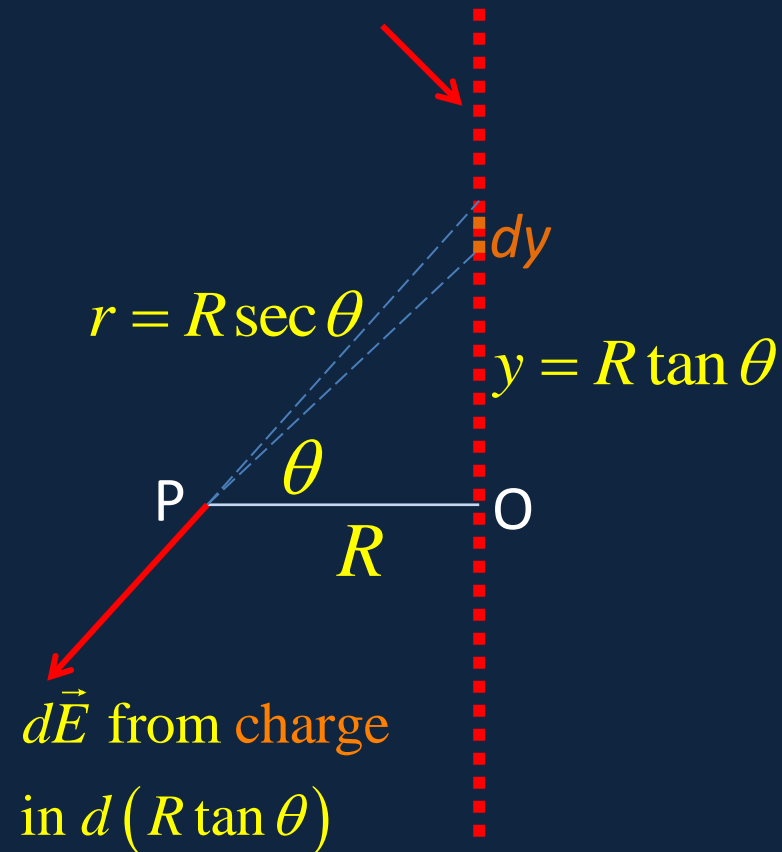
Electric Field from a *Plane* of Charge: Top View
Note: if you can't follow this, it doesn't matter!

Each red dot is a cross section of a wire perpendicular to the page.

- Imagine now we have a uniformly charged plane: we make it up of many parallel wires, each charge density λ, each perpendicular to the page, with *n* wires/meter.

- Remembering the field strength from a single wire is $k\lambda/2r$, and in *dy* there are *ndy* wires, the field strength at P from the charge in *dy* is:

$r = R\sec\theta$

dy

$y = R\tan\theta$

θ

P ———— O

R

$d\vec{E}$ from charge

in $d\left(R\tan\theta\right)$

$$\left|d\vec{E}\right| = \frac{2k\lambda n dy}{R\sec\theta} = \frac{2k\lambda n d\left(R\tan\theta\right)}{R\sec\theta} = 2k\lambda n\sec\theta d\theta$$

Electric Field from a *Plane* of Charge: Top View

- We've shown the field strength at P from the charged lines in *dy* is

$$\left|d\vec{E}\right| = \frac{2k\lambda n dy}{R\sec\theta} = \frac{2k\lambda n d\left(R\tan\theta\right)}{R\sec\theta} = 2k\lambda n\sec\theta d\theta$$
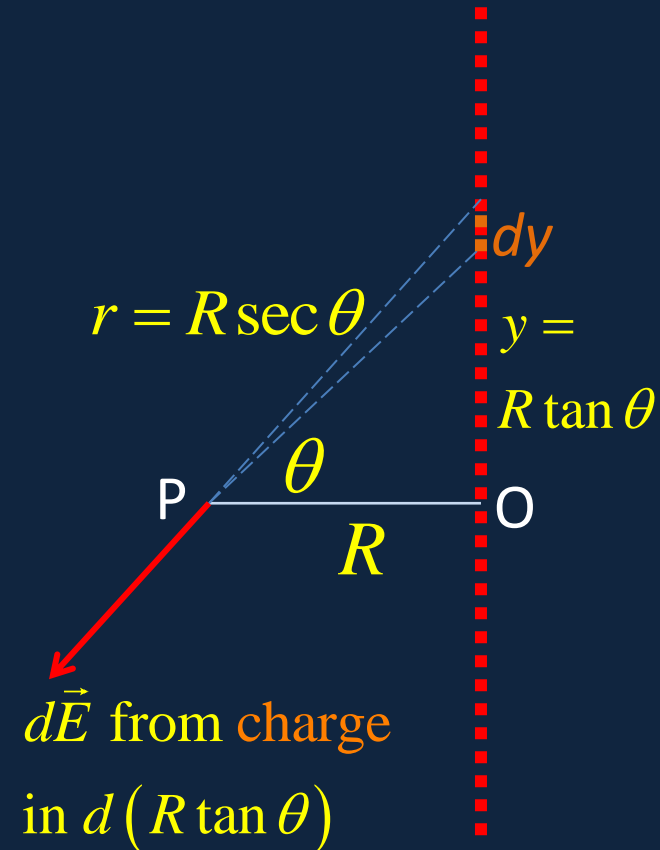
- This has component in the OP direction

$$\left|d\vec{E}\right|\cos\theta = 2k\lambda n d\theta$$

The total field is given by integrating,

$$E = 2k\lambda n\pi = 2k\sigma\pi = \frac{\sigma}{2\varepsilon_0}$$

where the plane charge density $\sigma = n\lambda$ Coulombs/m^2 , and $k = 1/4\pi\varepsilon_0$.

- **Notice the field strength is <u>constant</u>!**

$r = R\sec\theta$

dy

$y = R\tan\theta$

θ

P

R

O

$d\vec{E}$ from charge
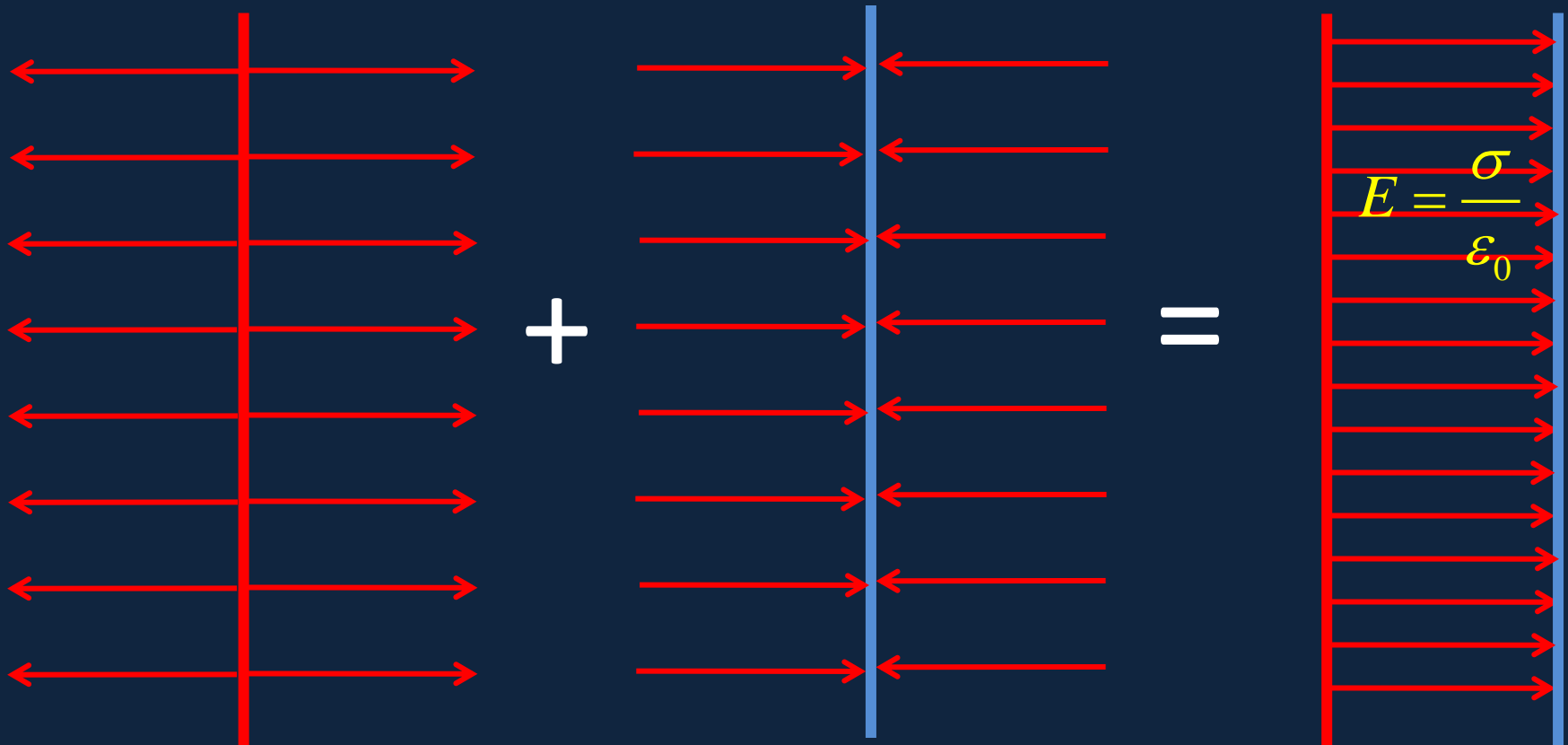
in $d\left(R\tan\theta\right)$

Electric Field from a Plane of Charge

- It's worth drawing the field lines to emphasize that the electric field from a uniformly charged plane is directly outward from the plane.

- For a finite plane of charge, this is a good approximation for distances from the plane small compared to the plane's extent.

$$E = 2k\sigma\pi = \frac{\sigma}{2\varepsilon_0}$$
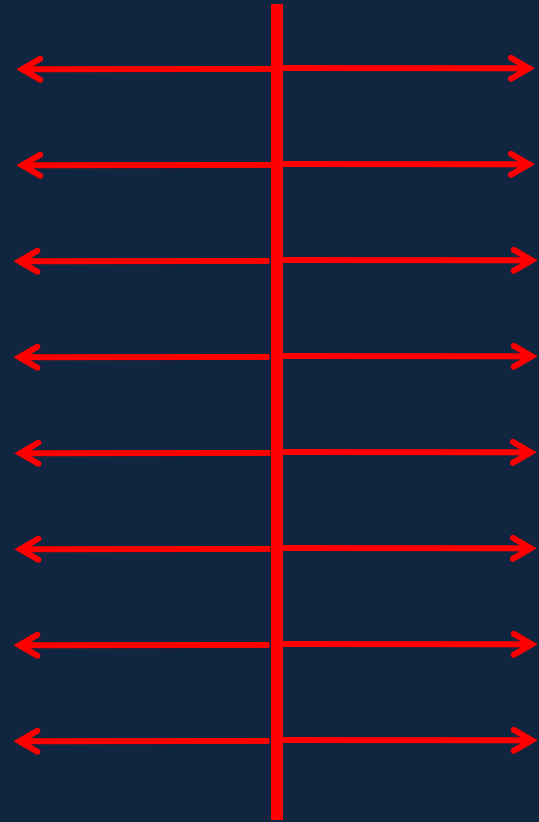
Field for Two Oppositely Charged Planes

$$E = \frac{\sigma}{\varepsilon_0}$$

Superpose the field lines from the negatively charged plate on the parallel positively charged one, and you'll see the total field is double in the space between the plates, but exactly <u>zero</u> outside the plates.