

More Statics

Physics 1425 Lecture 24

Statics: Conditions for Equilibrium

- For any body, $Md\vec{v}_{\text{CM}} / dt = \sum \vec{F}_i$, the net force causes the CM to accelerate. Hence, if the body is remaining at rest,

$$\sum_i \vec{F}_i = 0$$

- To eliminate *angular* acceleration, there must be zero torque about any axis. If all forces are in one plane, it's enough to prove zero torque about one axis perpendicular to the plane:

$$\sum_i \vec{\tau}_i = 0$$

Free Body Diagrams

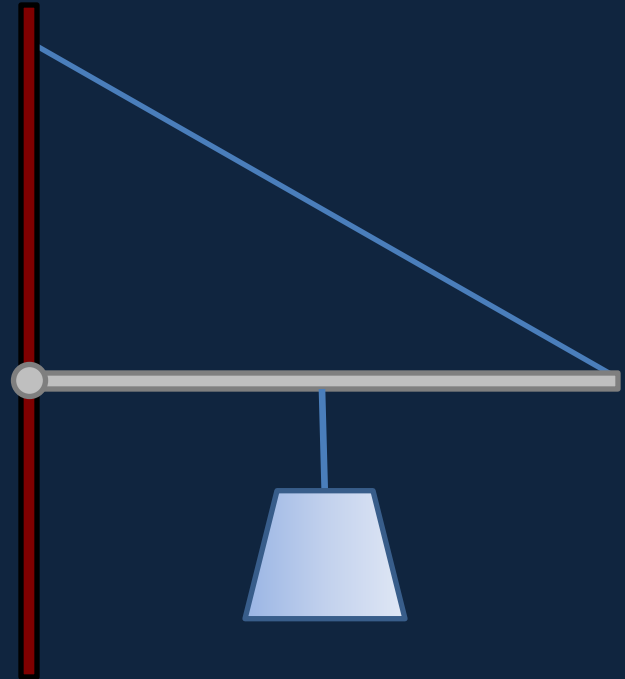
- To apply Newton's Laws to find how a body moves, we must focus on **that body alone** and add **all** the (vector) forces acting on it.
- The diagram showing all the forces on one body (or even part of a body) is called a "**free body diagram**"—we've "freed" the body from the rest of the system, representing everything else just by **the forces on this body**.
- The **net (total) force** then goes into $\Sigma \vec{F} = m\vec{a}$.

Tips about Torques

- In statics, the net torque of the forces on a body is zero about **any** point: choose a point one (or more) forces already pass through!
- The torque at the origin of a force \vec{F} acting at the point \vec{r} has magnitude rF_{\perp} , where F_{\perp} is the component of the applied force perpendicular to the line from the origin.

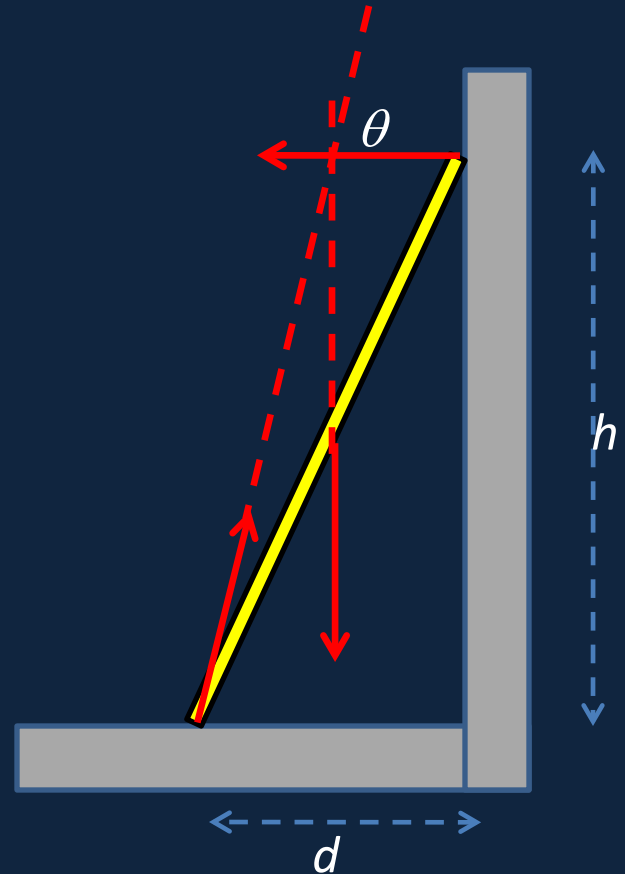
Clicker Question

- What is the approx tension T in the top string, given the mass is 2 kg, and it's hung from the midpoint of the rod, which is light and hinged, the angle is 30° ?
- A. 10 N
- B. 20 N
- C. $20\sqrt{3}$ N
- D. 40 N



Ladder Against a Smooth Wall

- Assume the wall exerts no significant friction, so the only force from the wall is the normal force.
- What angle does the floor force make with the horizontal?
- To ensure no torque, all three forces' lines of action must pass through the same point.
- Gravity acts at the midpoint, so $\tan \theta = h/0.5d$.



Warning: this trick saves working with components, but does NOT work if there are more than three forces! (Someone on the ladder.)

Window Washer's Nightmare?

- Is it possible for a ladder to rest against a wall in equilibrium if the wall is rough but the floor is smooth? (Not counting balanced vertically flat against the wall!).
 - A. Yes
 - B. No

Hooke's Law

- Hooke (same time as Newton) did many things: here he's holding a cord. (He'd found the shape, called a catenary, inverted, to be the ideal shape for constructing an arch.)
- **His Law:** for most materials under tension, the distance stretched is proportional to the force:

$$F = k\Delta\ell$$



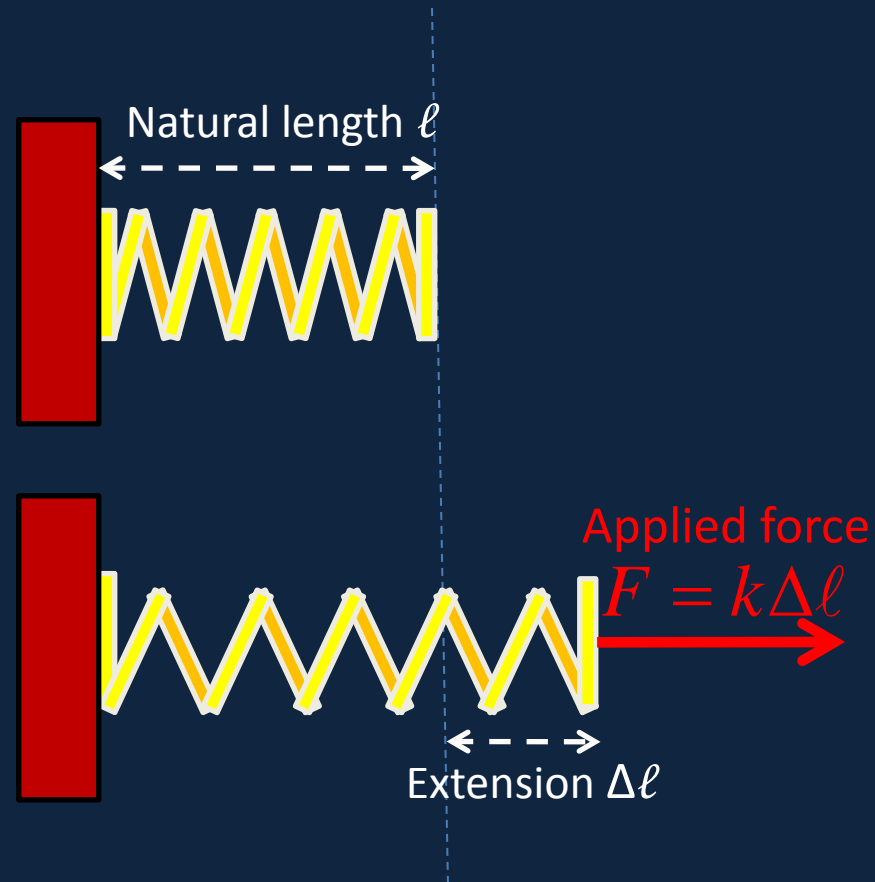
Review: Force on a Stretched Spring

- To extend a spring beyond its natural length by a distance $\Delta\ell$ takes a force

$$F = k\Delta\ell$$

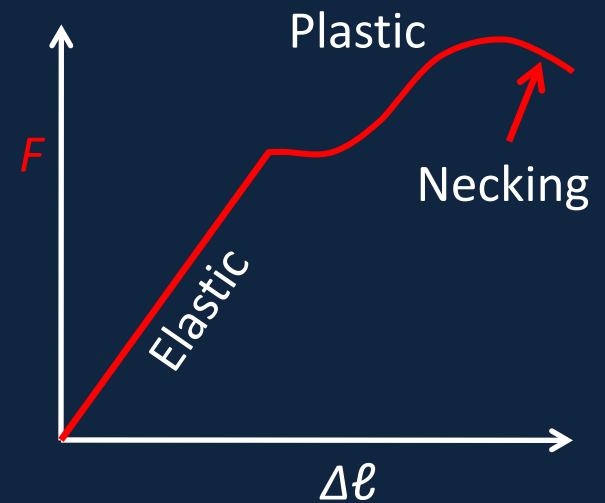
where k is the “spring constant”.

- This is **Hooke's Law**: and (as we've previously discussed) the spring pulls back with an equal but opposite force.



Hooke's Law $F = k\Delta\ell$

- Hooke's Law is not a fundamental Law, like Newton's laws: it's a summary of observations on a wide variety of materials.
- But everything has a breaking point—Hooke's Law works fine up to a force not far from the breaking force. Where it works is called the “**elastic region**”, if stretched this far the material **will spring back when released**. Just before breaking, it enters the “**plastic region**”—it stretches **irreversibly**.



The shape of the curve in the plastic region differs from one substance to another—just before breaking, narrowing often occurs at one place : “necking”.

Young's Modulus

- For a given tension, a wire **twice as long** will stretch **twice** as much—think of it as two wires end to end, under the same tension F .
- For a given applied force F , a wire with **twice the cross-section area** $2A$ will only stretch **half** as much—think of that as two identical parallel wires, each now carrying half the force.
- Young put these facts together to write:
$$\Delta\ell = \frac{1}{E} \frac{F}{A} \ell_0$$
- The **constant** E is called **Young's modulus**.