More Angular Momentum, then Statics

Physics 1425 Lecture 23
Vector Angular Momentum of a Particle

- A particle with momentum $\vec{p}$ is at position $\vec{r}$ from the origin O.
- Its angular momentum about the origin is\[ \vec{L} = \vec{r} \times \vec{p} \]
- This is in line with our definition for part of a rigid body rotating about an axis: *but also works for a particle flying through space.*

Viewing the x-axis as coming out of the slide, this is a “right-handed” set of axes:\[ \hat{i} \times \hat{j} = +\hat{k} \]
Clicker Question

• A particle moves along a straight line at constant speed. The line does not pass through the origin. Is the particle’s angular momentum about the origin constant?

A. Yes
B. No
Rotational Motion of a Rigid Body

• For a collection of interacting particles, we’ve seen that

\[ \frac{d\vec{L}}{dt} = \sum_i \vec{\tau}_i \]

the vector sum of the applied torques, \( \vec{L} \) and the \( \vec{\tau}_i \) being measured about a fixed origin \( O \).

• A rigid body is equivalent to a set of connected particles, so the same equation holds.

• It is also true (proof in book) that even if the CM is accelerating,

\[ \frac{d\vec{L}_{CM}}{dt} = \sum \vec{\tau}_{CM} \]
A dumbbell (two small masses at the ends of a light rigid rod) is mounted on a fixed axle through its center, at an angle $\theta$. It is set in steady rotation. The direction of the angular momentum of the system is:

A. Along the axle
B. Along the dumbbell rod
C. Neither of the above.
A Bit More About $\vec{L}$ and $\vec{\omega}$...

- We’ve used $\vec{L} = I \vec{\omega}$ a lot.
- We see from this example it’s not always true that $\vec{L}, \vec{\omega}$ are parallel vectors.
- What’s going on?
- The answer is that $\vec{L}, \vec{\omega}$ are only parallel if the spinning body is symmetric about the axis of rotation—which is usually the case.
- For more complicated cases, you will still see $\vec{L} = I \vec{\omega}$, but that fat $I$ denotes a tensor or matrix.
Spinning Top

- Pointing your right thumb in the direction of the angular velocity vector $\vec{\omega}$, your curling fingers point in the direction of rotation.
- Gravity exerts a torque about the pivot point $\vec{\tau} = \vec{d} \times m\vec{g}$, evidently directed inwards.
- From $\vec{\tau} = d\vec{L} / dt = Id\vec{\omega} / dt$ $d\vec{\omega}$ will be inwards, the tip of $\vec{\omega}$ is describing a horizontal circle: this is “precession”.

\[ \tau = \vec{d} \times m\vec{g} \]
Precession Rate

The horizontal component of the angular velocity vector \( \vec{\omega} \) has length \( \omega \sin \phi \) and it precesses around a circle centered above the pivot point.

The precession angular velocity is written \( \Omega = d\theta / dt \), where \( \theta \) measures angle around the horizontal circle.

If in time \( dt \) there is precession through \( d\theta \),

\[
d\omega = \left( \omega \sin \phi \right) d\theta
\]

so

\[
\frac{d\theta}{dt} = \frac{1}{\omega \sin \phi} \frac{d\omega}{dt} = \frac{1}{\omega \sin \phi} \frac{\tau}{I} = \frac{mgd}{\omega I}
\]

\[
\vec{\tau} = \frac{d\vec{L}}{dt} = I d\vec{\omega} / dt
\]
Statics: Conditions for Equilibrium

• For any body, \( \frac{Md\ddot{v}_{CM}}{dt} = \sum \vec{F}_i \), the net force causes the CM to accelerate. Hence, if the body is remaining at rest,

\[
\sum_i \vec{F}_i = 0
\]

• To eliminate angular acceleration, there must be zero torque about any axis. If all forces are in one plane, it’s enough to prove zero torque about one axis perpendicular to the plane:

\[
\sum_i \vec{\tau}_i = 0
\]
Free Body Diagrams

• To apply Newton’s Laws to find how a body moves, we must focus on the body alone and add all the (vector) forces acting on it.

• The diagram showing all the forces on one body (or even part of a body) is called a “free body diagram”—we’ve “freed” the body from the rest of the system, representing everything else just by the forces on this body.

• The net (total) force then goes into \( \Sigma \vec{F} = m\vec{a} \).
Flat Forces?

• If a body in equilibrium is acted on by three and only three forces, do the force vectors have to lie in a plane?

A. Yes
B. No
Flat Forces

- If just three forces are acting on a body, and it’s in equilibrium, they must all lie in the same plane, because if we choose the plane defined by two of them, and the third force has a component perpendicular to that plane, nothing is balancing this perpendicular force.
Clicker Question

• A body is in equilibrium. It is acted on by three forces, lying in a plane.
• Do the lines of action of the three forces all pass through the same point?

A. Yes
B. No
Three Force Equilibrium

- If a body is in equilibrium when acted on by three forces, the three forces must lie in the same plane AND all pass through a common point. If they don’t, taking a perpendicular axis through a point where two of them meet, the third force gives an unbalanced torque about that point, so the body will have angular acceleration.
Clicker Question

• What is the tension $T$ in the horizontal string?

A. $Mg \cos \theta$
B. $Mg \tan \theta$
C. $Mg \cot \theta$
D. None of the above.
Clicker Question

• What is the approx tension $T$ in the top string, given the mass is 2 kg, and it’s hung from the midpoint of the rod, which is light and hinged, the angle is 30°?

A. 10 N
B. 20 N
C. $20\sqrt{3}$ N
D. 40 N