Center of Mass

Physics 1425 Lecture 17

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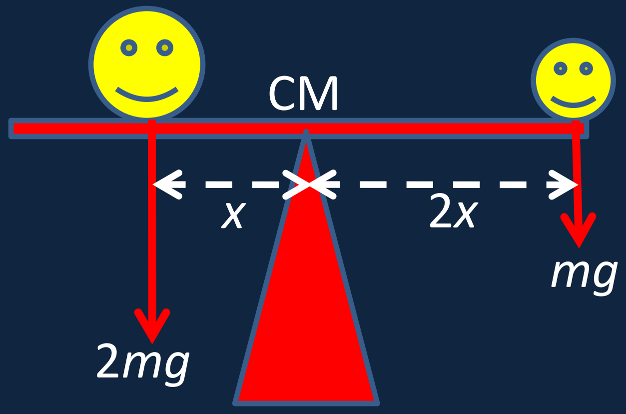
Center of Mass and Center of Gravity

- Everyone knows that if one kid has twice the weight, the other kid must sit twice as far from the axle to balance.
- Each kid then has the same torque about the axle:
- **Torque = force x distance from the axle of the force's line of action.**
- The gravitational forces balance about the axle: it's at the center of gravity—aka the center of mass.



Center of Mass in One Dimension

- Recall the center of mass of two objects is defined by

$$\left(m_1 + m_2\right) x_{\mathrm{CM}} = M x_{\mathrm{CM}} = m_1 x_1 + m_2 x_2$$

- Notice that if we take x_{CM} as the origin (the center of mass frame) then the equation is just

$$m_1 x_1 + m_2 x_2 = 0$$

precisely the balance equation from before (one of those *x*'s is negative, of course).

CM of Several Objects in One Dimension

- The general formula is:

$$x_{\mathrm{CM}} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i} = \frac{\sum_{i=1}^{n} m_i x_i}{M}$$

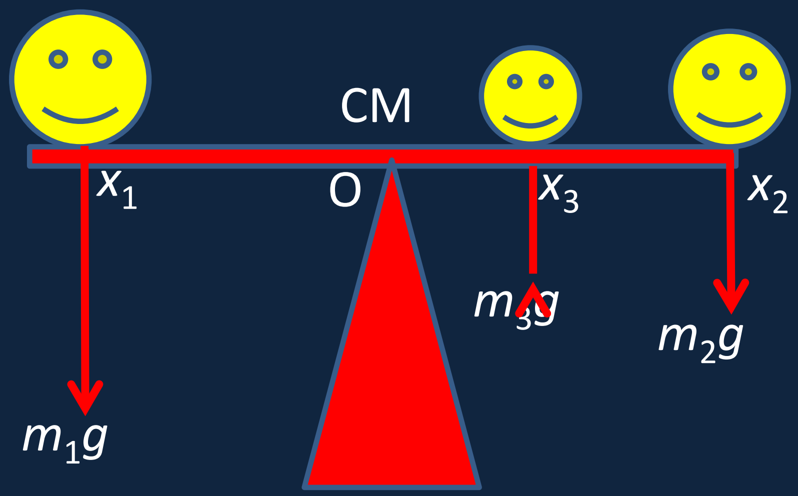
- But before putting in numbers, it’s worth staring at the system to see if it’s symmetric about any point!

Add Another Kid to the Seesaw…

- For the three to be in balance, the sum of the torques about the axle must be zero, so:

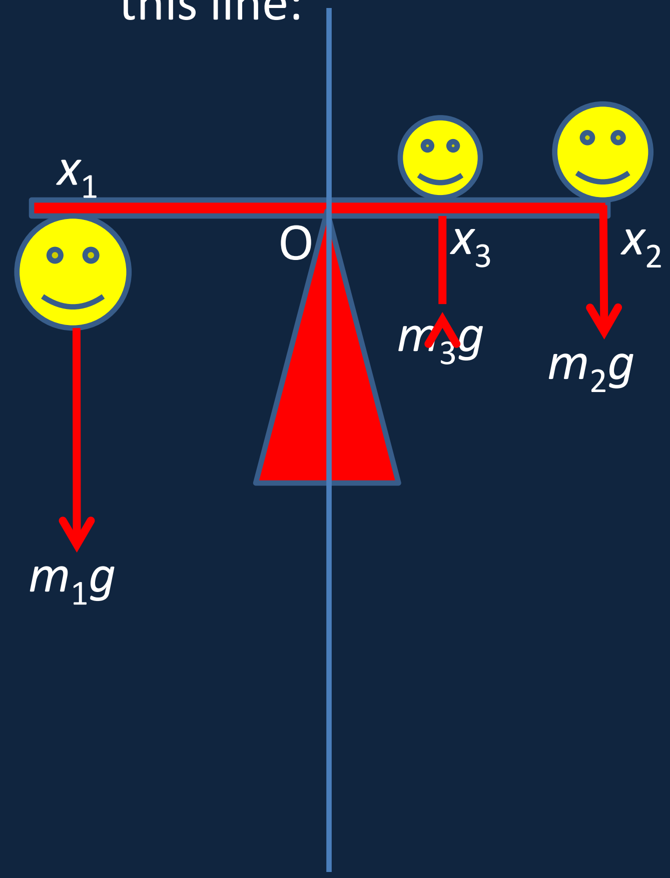
$$m_1x_1 + m_2x_2 + m_3x_3 = 0$$

- That is to say, the *x* coordinate of the center of mass must be the same as the *x*-coordinate of the axle.
- This is clearly extendable to *any* number of masses ….



Some Gymnastics

- The equation
  $$m_1 x_1 + m_2 x_2 + m_3 x_3 = 0$$
  is still correct even if one kid is hanging by his hands below the seesaw!
- The center of mass is not *at* the balance point (the axle) but *is in the same vertical straight line*.



Center of Mass of a Two-Dimensional Object

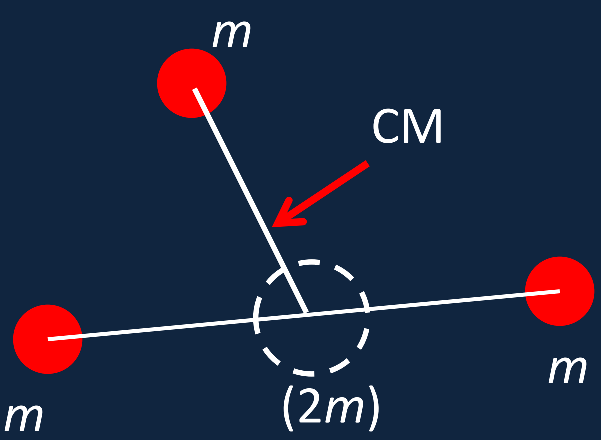
- Think of some shape cut out of cardboard.
- Hang it vertically by pushing a pin through some point.
- Think of it as made up of many small masses—when it's hanging at rest, the center of mass will be somewhere on the vertical line through the pin. <u>Draw the line.</u>
- Repeat with the pin somewhere else: the lines you drew meet at the CM.

Tip: if the object is symmetric about some line, the center of mass will be *on that line*!

Three Equal Masses

- If we have three equal masses at the corners of a triangle, the center of mass of two of them is the half-way point on the side joining them.
- We can replace them by a mass $2m$ at that point, then the CM of *all three masses* is on the line from the other vertex to that point, one-third of the way up.
- This is the centroid of the triangle, and is at

$$\vec{r} = \frac{\vec{r}_1 + \vec{r}_2 + \vec{r}_3}{3}$$

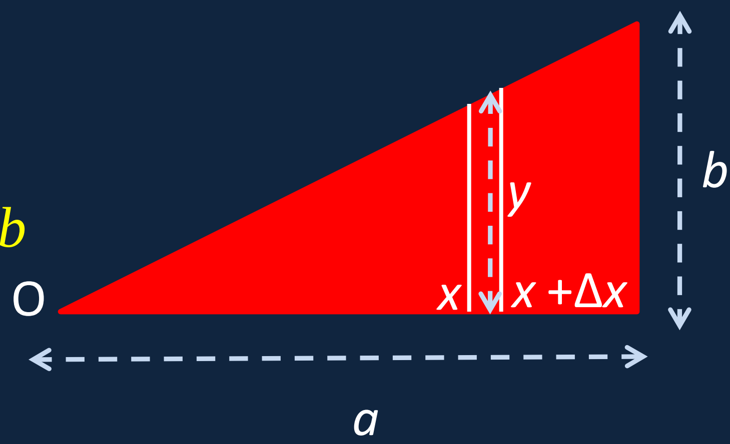


Center of Mass of a Solid Triangle

- We'll take a right-angled triangle. The *x*-coordinate of the CM is found by the integral generalization of the sum $$Mx_{\text{CM}} = \sum_{i=1}^{n} m_i x_i$$
- If the triangle has area mass density ρ kg/m^2, the strip shown has mass $\rho y \Delta x$, and $M = ½\, \rho ab$, so

$$\tfrac{1}{2}\rho abx_{\text{CM}} = \int_0^a \rho xydx = (b/a)\int_0^a \rho x^2 dx = \tfrac{1}{3}\rho a^2 b$$

from which the CM is at (2/3)*a*.



- Bottom line: the CM of the solid triangle is at the same point as the CM of three equal masses at the corners!

The height *y* of the strip at *x* is given by *y*/*b* = *x*/*a*, from similar triangles.