More about Momentum

Physics 1425 Lecture 15

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Elastic One-Dimensional Collisions 1

- <u>An elastic collision is one in which mechanical energy is conserved.</u>
- First example: two equal masses with opposite velocities:

- Conservation of *momentum* tells us they bounce back with equal speeds, and conservation of *energy* ensures each ball just has its velocity reversed.
- After:

Clicker Question

Suppose we have two equal masses, one initially at rest, the other approaching from the left at velocity *v*. What is the velocity of the *center of mass*?

A. 0
B. 0.5*v*
C. *v*
D. Something else

Clicker Question

What is the velocity of the center of mass after the elastic collision?

A. 0

B. 0.5v

C. -0.5v

D. Something else

Clicker Question

What would be the velocity of the center of mass after a completely <u>inelastic</u> collision? (Assume the balls stick together.)

A. 0

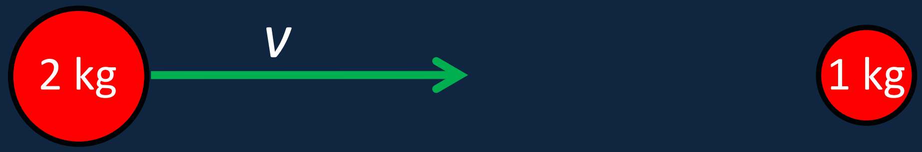
B. 0.5v

C. -0.5v

D. Something else

Clicker Question

Suppose we have a 1 kg mass initially at rest, and a 2 kg mass approaching from the left at velocity *v*. What is the velocity of the center of mass?



A. 0.2*v*

B. 0.33*v*

C. 0.5*v*

D. 0.67*v*

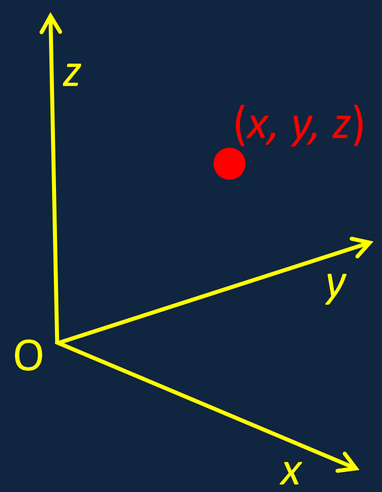
E. *v*

Looking at Collisions…

- Collisions of two objects, in one or two dimensions, are often easier to understand (as you'll see) if we examine the *motions relative to the center of mass*.
- This is called working in the center of mass frame of reference.
- If there are no external forces acting on the system, overall momentum is conserved, and the center of mass moves at constant velocity relative to any given inertial frame.

Reminder: a Frame of Reference

Frame of reference:



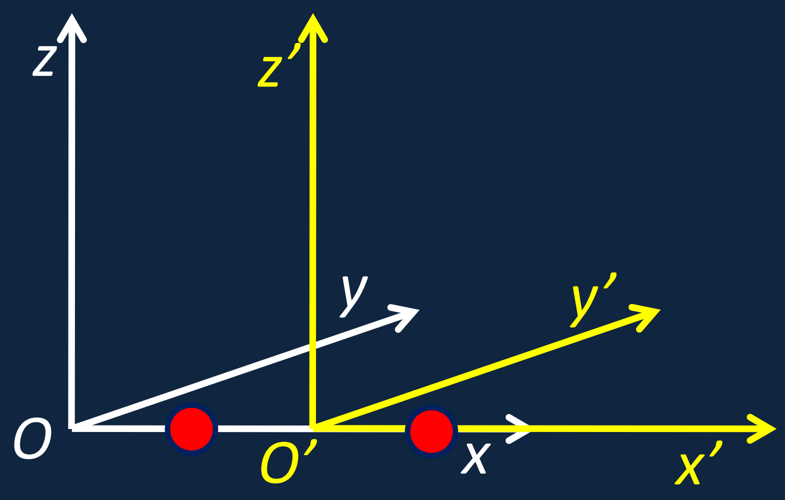
The frame can be envisioned as three meter sticks at right angles to each other, like the beginning of the frame of a structure.

To measure motion, we must first measure position.

We measure position relative to some fixed point O, called the origin.

We give the ball's location as (x, y, z): we reach it from O by moving x meters along the x-axis, followed by y parallel to the y-axis and finally z parallel to the z-axis.

Two Inertial Frames of Reference

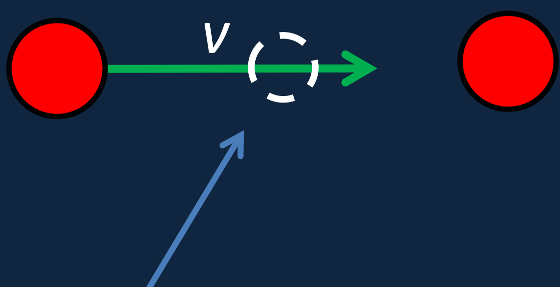


The fixed lab frame is in white.
The CM frame is in yellow. The origin *O'* is always at the center of mass of the two balls. With no external forces, *O'* moves at constant velocity relative to *O*, so both frames are inertial, both conserve momentum.

- Suppose the first frame, call it the lab frame, is three sticks fixed on the bench.
- Our experiment is an elastic collision of two balls, no outside forces.
- The second frame, the CM frame, has its origin at the center of mass of the two balls. It's moving at a *constant velocity* relative to the lab frame.

Equal Masses, Two Frames

- **Lab frame:**
- Before:



CM has velocity $v/2$ in lab frame.

- After: just add the CM velocity of $v/2$ to the velocities in the CM frame!

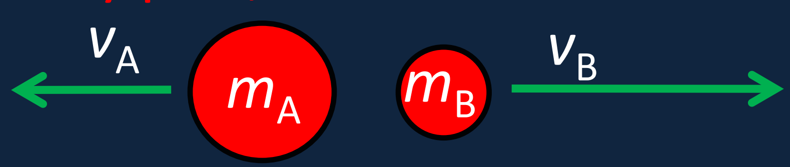
- **CM Frame:**
- Before:

- After:

Elastic One-Dimensional Collisions

- Third example: two <u>unequal</u> masses <u>in the CM frame</u>:

- <u>Total momentum $\vec{P} = M\vec{v}_{CM}$ is always zero in the CM frame</u>, so they have equal but opposite momenta. As they collide, the equal elastic forces reduce both momenta to zero at the same instant—then supply *exactly the same forces in reverse* as the balls spring back.
- Therefore, as they part, each ball has its initial velocity reversed.

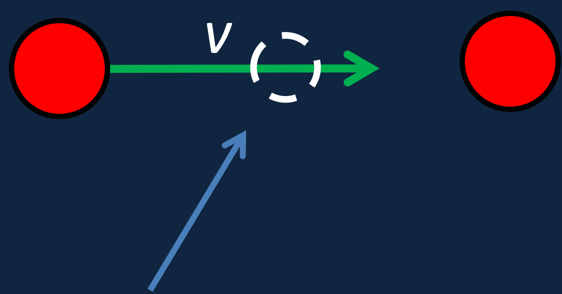


Elastic One-Dimensional Collisions

- What about unequal masses not in the center of mass frame of reference?
- The simplest approach is to first find the velocity of the center of mass, then the velocities of the two particles relative to the center of mass, then realize that after the collision, these relative velocities will have been reversed as we just discussed.
- In other words, solve the problem in the center of mass frame, then translate back!

Equal Masses, Two Frames, Inelastic

- **Lab frame:**
- Before:

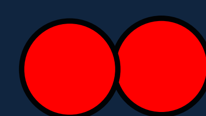


CM has velocity $v/2$ in lab frame.

- After: just add the CM velocity of $v/2$ to the velocities in the CM frame!

- **CM Frame:**
- Before:

- After: stuck!



Clicker Question

For the one-dimensional inelastic collision of a mass m moving at velocity v hitting and sticking to an initially stationary mass m, is the kinetic energy loss:

A. Greater in the center of mass frame?
B. Greater in the lab frame (one mass initially at rest)?
C. The same in both?

Clicker Question

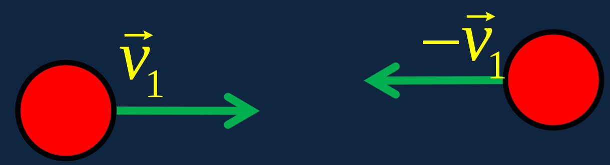
I drop a large ball with a small ball balanced on top of it from a height of one meter. The small ball stays on top during the fall. After the large ball bounces off the floor, how high do you predict the small ball will go?

A. 1 meter
B. 2 meters
C. 3 meters
D. 4 meters or more

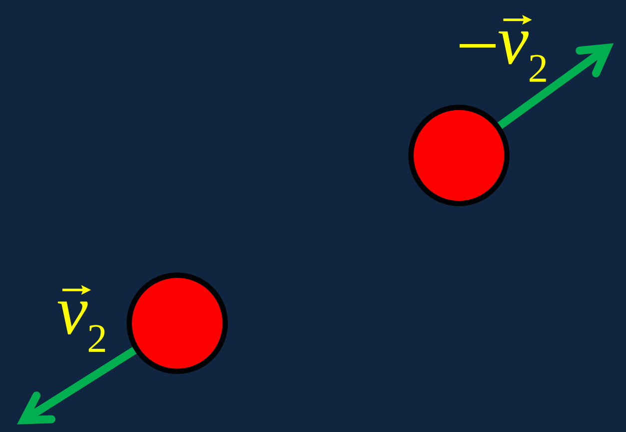
Two-Dimensional CM Elastic Collisions

- As in one dimension, in the CM frame *the total momentum is zero throughout*—BUT now that is not enough to solve the problem.
- The balls could come out at a different angle, as any pool player knows.
- <u>Animation!</u>

- Before:



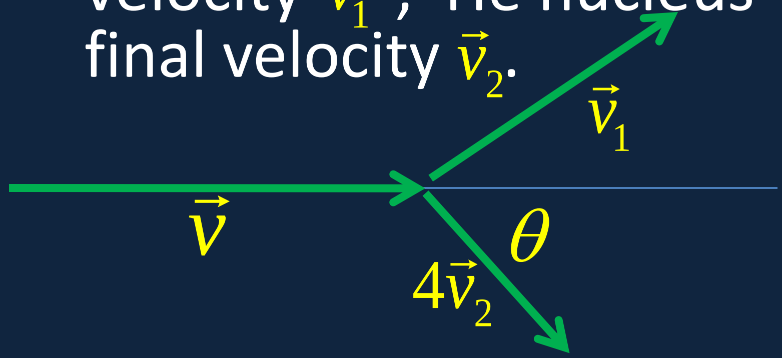
- After:



Problem from Book

- A neutron collides elastically with a helium nucleus (at rest initially) whose mass is four times that of the neutron.
- The helium nucleus is observed to move off at an angle θ.
- Determine the angle of the neutron, and the speeds of the two particles, and after the collision. The neutron's initial velocity is $\vec{v}$.

- Take final neutron velocity $\vec{v}_1$, He nucleus final velocity $\vec{v}_2$.



- Conservation of momentum:

$$\vec{v} = \vec{v}_1 + 4\vec{v}_2$$

- Conservation of energy:

$$\vec{v}^{\,2} = \vec{v}_1^{\,2} + 4\vec{v}_2^{\,2}$$

- Go to next slide…

Solving the problem…

- We have momentum conservation and energy conservation equations:

$$\vec{v} = \vec{v}_1 + 4\vec{v}_2 \qquad\qquad \vec{v}^{\,2} = \vec{v}_1^{\,2} + 4\vec{v}_2^{\,2}$$

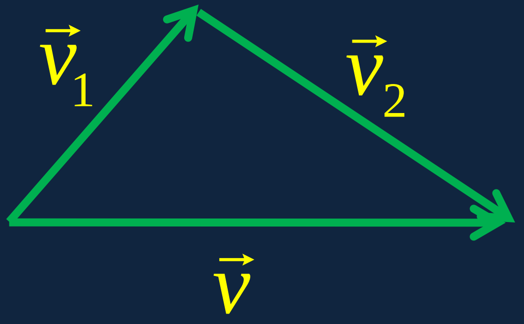
- Important! The only angle we know is that between $\vec{v}, \vec{v}_2$.
- The strategy is to eliminate one variable by choosing one in the momentum equation to put in the energy equation.
- We choose $\vec{v}_1$ *because* we can square $\left(\vec{v} - 4\vec{v}_2\right)$: *we know the angle between these two vectors*.
- This gives $\vec{v}^{\,2} = \left(\vec{v} - 4\vec{v}_2\right)^2 + 4\vec{v}_2^{\,2} = \vec{v}^{\,2} - 8vv_2\cos\theta + 20v_2^2$
- from which immediately $v_2 = 0.4v\cos\theta$.

Another Book Problem…

- **61.** (III) Prove that in the elastic collision of two objects of identical mass, with one being a target initially at rest, the angle between their final velocity vectors is always 90°.

- All masses are equal—so momentum vectors are just velocity vectors multiplied by *m*, let's say *m* = 1.
- Initial velocities $\vec{v}, 0$.
- Final velocities $\vec{v}_1, \vec{v}_2$.

$$\vec{v} = \vec{v}_1 + \vec{v}_2, \quad v^2 = v_1^2 + v_2^2$$



- From Pythagoras' theorem, this triangle of the velocities is a *right angle* triangle!

Check this with the animation