Momentum

Physics 1425 Lecture 15

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Physics Definition of Momentum

- Momentum is another word (like work, energy, etc.) from everyday life that has a precise meaning when used in physics.
- To begin with, we discuss point particles (or small enough bodies they can be considered points). We'll get to bigger things soon.
- The momentum of a particle of mass *m* moving with velocity $\vec{v}$ is written

$$\boxed{\vec{p} = m\vec{v}}$$

Momentum and Newton’s Second Law

- We’ve written Newton’s Second Law as

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

- In fact Newton wrote it

$$\vec{F} = \frac{d}{dt}m\vec{v} = \frac{d\vec{p}}{dt}$$

- (of course, in a different notation).
- This difference becomes important in relativity—*nothing can be accelerated beyond the speed of light*, near that speed an applied force will cause an increase in the mass of an object.

Momentum and Newton's *Third* Law

- If two particles are interacting, Newton's Third Law tells us the force from *A* on *B* and from *B* on *A* are equal and opposite:

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

- Assuming for the moment that no other forces are present, the two momenta change at rates

$$\vec{F}_{AB} = \frac{d\vec{p}_B}{dt}, \quad \vec{F}_{BA} = \frac{d\vec{p}_A}{dt}$$

- From which

$$\frac{d}{dt}\left(\vec{p}_A + \vec{p}_B\right) = 0$$

- Total momentum does not change: it is conserved.

Lots More Particles....

- Suppose we have a large number of particles, interacting with each other with forces $\vec{F}_{mn}^{\text{int}}$, and also acted on by external forces, like gravity or electric fields.
- One of the particles will have rate of change of momentum
$$\frac{d\vec{p}_n}{dt} = \vec{F}_n^{\text{ext}} + \sum_{m \neq n} \vec{F}_{mn}^{\text{int}}$$
- If we add together the equations for all the particles, the internal forces cancel in pairs, leaving
$$\frac{d\vec{P}}{dt} = \sum_n \frac{d\vec{p}_n}{dt} = \sum_n \vec{F}_n^{\text{ext}}$$
- The total momentum is <u>only</u> changed by <u>external</u> forces.

Impulsive Force

- A large force operating for a very short time is often termed an *impulse*.
- If the force $\vec{F}$ operates for a time Δt, the impulse

$$\vec{J} = \vec{F}\Delta t$$

- Impulsive forces usually vary rapidly with time (as when a bat hits a ball), and then

$$\vec{J} = \int \vec{F}(t)\,dt$$

- An impulsive force causes *a change in momentum equal to the impulse*:

$$\vec{p}_{\text{final}} - \vec{p}_{\text{initial}} = \int \frac{d\vec{p}}{dt}dt = \int \vec{F}dt = \vec{J}$$

Clicker Question

Two balls of putty of equal mass approach each other from opposite directions at equal speeds. They stick together and come to rest.
Was momentum conserved in this collision?

A. Yes
B. No

Clicker Question

A pendulum consists of a wooden ball hanging motionless on a string. A bullet is shot horizontally, hitting the pendulum head-on on its equator. The bullet bounces back, the pendulum swings. On a second attempt, after the pendulum is again at rest, the bullet penetrates and stays in the wood. Which caused the bigger swing?

A. The pendulum swung more when the bullet bounced off
B. The pendulum swung more when the bullet stayed with it

Clicker Question

I drop a hard rubber ball on to the floor from a height of one meter. As it bounces, it is squashed 1 cm at maximum. *Very approximately*, what is the force it feels from the floor at the moment in the middle of the bounce when it is at rest?

A. mg
B. 5 mg
C. 10 mg
D. 25 mg
E. 100 mg

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Impulsive forces are big! The velocity v gained in falling 1 meter was lost in 1 cm. From $v^2 = 2ax$, if we take the deceleration on hitting the floor to be constant, it is about 100g. This is an approximation, but in the right ballpark.

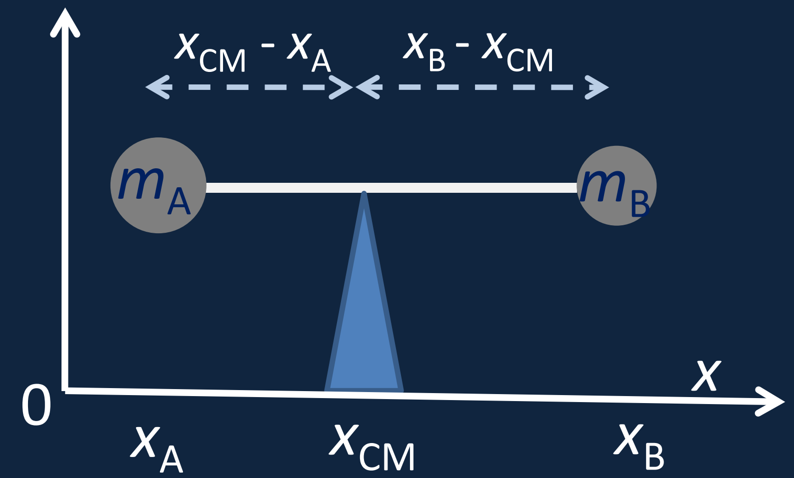
Center of Mass of Two Particles

- If the two particles are at the ends of a light rod, their center of mass x_{CM} is the point about which they would balance:

$$m_A\left(x_{CM} - x_A\right) = m_B\left(x_B - x_{CM}\right)$$

and from this

$$x_{CM} = \frac{m_A x_A + m_B x_B}{m_A + m_B}$$



If the rod isn't parallel to the x-axis, we need the **three-dimensional** version:

$$\vec{r}_{CM} = \frac{m_A \vec{r}_A + m_B \vec{r}_B}{m_A + m_B}$$

Center of Mass and Total Momentum

- For two particles, writing the total mass

$$M = m_{\mathrm{A}} + m_{\mathrm{B}}$$

the center of mass is given by

$$M\vec{r}_{\mathrm{CM}} = m_{\mathrm{A}}\vec{r}_{\mathrm{A}} + m_{\mathrm{B}}\vec{r}_{\mathrm{B}}$$

and differentiating to find its time dependence

$$M\vec{v}_{\mathrm{CM}} = m_{\mathrm{A}}\vec{v}_{\mathrm{A}} + m_{\mathrm{B}}\vec{v}_{\mathrm{B}} = \vec{p}_{\mathrm{A}} + \vec{p}_{\mathrm{B}} = \vec{P}$$

Bottom line: the total momentum of the system equals the total mass multiplied by the CM velocity.

Motion of the Center of Mass

- We saw earlier that the *total* momentum of a system is only changed by external forces:

$$\frac{d\vec{P}}{dt} = \sum_n \frac{d\vec{p}_n}{dt} = \sum_n \vec{F}_n^{\text{ext}}$$

- We now see that $\vec{P} = M\vec{v}_{\text{CM}}$.
- *It follows that the motion of the center of mass is as if* all *the mass were concentrated there, and* all *the external forces acted there.*
- For zero external forces, $\vec{v}_{\text{CM}}$ is constant.