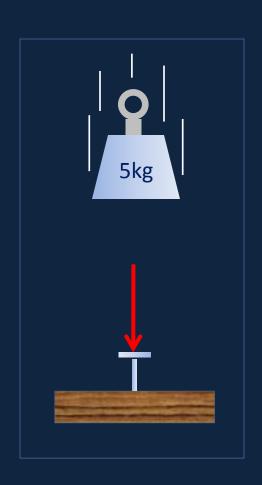
Kinetic Energy and Energy Conservation

Physics 1425 Lecture 13

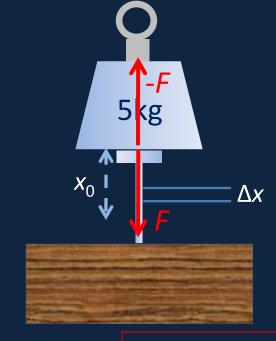
Moving Things Have Energy

- Energy is the ability to do work: to deliver a force that acts through a distance.
- Placing a weight gently on a nail does nothing.
- Dropping the weight on the nail can drive the nail into the wood.
- If the weight is moving when it hits the nail, it has the ability to do work driving the nail in. This is its *kinetic* energy.



How Much Work Does the Moving Weight Do?

- After contact with the nail, the forces between the weight and the nail are equal and opposite. Suppose the nail is driven in a total distance x_0 .
- In going through a small distance Δx , the work done on the nail $\Delta W = F \Delta x$.
- Meanwhile, for the weight -F = ma, the weight has slowed down: $-F = m\Delta v/\Delta t$.
- Therefore $\Delta W = F\Delta x = -m \Delta v \Delta x / \Delta t$.
- Now for small Δx , we can take $\Delta x/\Delta t = v$, so $\Delta W = -mv\Delta v$, and x_0



Weight hits nail with speed v_0

$$W = \int_{0}^{x_0} dW = -\int_{0}^{0} mv dv = \frac{1}{2} mv_0^2$$

Where Did the Weight's Energy Come From?

- We've seen that if the weight hits the nail and comes rapidly to rest, it loses energy $\frac{1}{2} \frac{mv_0^2}{n^2}$.
- This is its kinetic energy K at speed v_0 .
- Let's suppose it gained that energy by being dropped from rest at a height h.
- At uniform acceleration g, $v_0^2 = 2gh$.
- So the kinetic energy $\frac{1}{2}mv_0^2 = mgh$: precisely the potential energy lost in the fall—the work done by gravity $mgh = force mg \times distance h$.

A Small Kinetic Energy Change

• Suppose the velocity of a mass m changes by a tiny amount $\Delta \vec{v}$ as the mass moves through $\Delta \vec{r}$. Then the change in kinetic energy K is (dropping the *very* tiny $(\Delta v)^2$ term)

$$\Delta K = \frac{1}{2}m(\vec{v} + \Delta \vec{v})^2 - \frac{1}{2}m\vec{v}^2 = m\vec{v} \cdot \Delta \vec{v} = m\frac{\Delta \vec{r} \cdot \Delta \vec{v}}{\Delta t}$$

• Note this depends only on the change in speed—the dot product ensures that only the component of $\Delta \vec{v}$ in the direction of \vec{v} counts. The displacement $\Delta \vec{r}$ is of course in direction \vec{v} .

Energy Balance for a Projectile

- Consider a projectile acted on only by gravity, moving a distance $\Delta \vec{r}$ in a short time Δt .
- Gravity does work $m\vec{g} \cdot \Delta \vec{r} = -\Delta U$, where U is the gravitational potential energy.
- The change in velocity $\Delta \vec{v} = \vec{g} \Delta t$, so the change in potential energy

$$\Delta U = -m\vec{g} \cdot \Delta \vec{r} = -m \frac{\Delta \vec{v} \cdot \Delta \vec{r}}{\Delta t} = -\Delta K!$$

• The total energy *U* + *K* does not change.

Conservation of Mechanical Energy

 We've established that for a projectile acted on only by gravity K.E. + P.E. = a constant,

$$\frac{1}{2}m\vec{v}^2 + mgh = E,$$

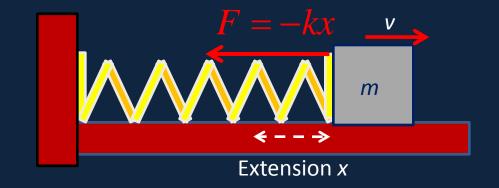
- Here E is called the total (mechanical) energy.
- This is valid if:
- A. We can neglect air resistance, friction, etc.
- B. Other forces acting are always perpendicular to the direction of motion: so this will also be true for a roller coaster, ignoring friction.

Springs Conserve Energy, Too

- Suppose the spring is fixed to the wall, at the other end a mass m slides on a frictionless surface.
- By an exactly similar argument to that for gravity, we can show

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = E,$$

constant total energy.



Conservative and Nonconservative Forces

- Gravity and the spring are examples of conservative forces: if work is done against them, they store it all as potential energy, and it can be used later. Total mechanical energy is conserved.
- Friction is not a conservative force: work done against friction generates heat, it does not conserve the mechanical energy, little of which can be recovered.

Different Paths for a Conservative Force

- For a conservative force, suppose taking an object from point A to point B along path P₁ requires us to supply work W₁. Then if we let the object slide back from B to A, the force will fully reimburse us, giving back all the work W₁.
- Now suppose there's another path P₂ from A to B, and using that path takes less work from us, W₂.
- We can construct a track going from A to B along P₂ then back along P₁, and we'll gain energy! This is a perpetual motion machine...so what's wrong?

Potential Energy in a Conservative Field

• Imagine a complicated conservative field, like gravity from Earth + Moon at any point. We've established that the work we need to do to take a mass m from point \vec{r}_A to \vec{r}_B ,

$$W(\vec{r}_A, \vec{r}_B) = \int_{\vec{r}_A}^{\vec{r}_B} \vec{F} \cdot d\vec{r}$$

depends *only* on the endpoints, <u>not the path</u>—so we can unambiguously define a potential energy difference

$$U(\vec{r}_B) - U(\vec{r}_A) = \int_{\vec{r}_A}^{\vec{r}_B} \vec{F} \cdot d\vec{r}$$

Potential Energy Determines Force

- If we know the potential energy $U(\vec{r})$ in a complicated gravitational field, how can we find the gravitational force on a mass m at \vec{r} ?
- Take a very short path going in the x-direction:

$$U(\vec{r} + \Delta x) - U(\vec{r}) = \int_{\vec{r}}^{\vec{r} + \Delta x} \vec{F} \cdot d\vec{r} = F_x \Delta x$$

• We must apply a force F_x to move this small distance, so the opposing gravitational force is given by $F_{Gx} = -\partial U(\vec{r})/\partial x$.

More on Potential Energy and Force

- Since the potential energy is given by integrating the force through a distance, it's not surprising that we get back the force by differentiating the potential energy.
- For gravity near the Earth's surface, $U(\vec{r}) = mgz$, taking z as vertically up, so $F_{\tau} = -\partial U / \partial z = -mg$
 - and since *U* doesn't depend on *x* or *y*, there is no force in those directions.
- Reminder! Forces and work depend only on changes in potential energy—we can set the zero of potential energy wherever is convenient, like ground level.

Potential Energy and Force for a Spring

For a spring,

$$U(x) = \frac{1}{2}kx^2$$

a parabola.

The force the spring exerts when extended to *x*

$$F(x) = -dU(x)/dx = -kx$$

It's worth staring at the U(x) graph, bearing in mind that the force at any point is the negative of the slope there—see how it gets steeper further away from the origin.

