Work and Energy

Physics 1425 Lecture 12

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What is Work and What Isn't?

- In physics, work has a very restricted meaning!
- *Doing homework isn't work.*

- Carrying somebody a mile on a level road isn't work…

- Lifting a stick of butter three feet *is* work—in fact, about one unit of work.

Work is *only* done by a *force*…

- and, the force has to move something!
- Suppose I lift one kilogram up one meter…
- I do it at a slow steady speed—my force just balances its weight, let’s say 10 Newtons.

- **Definition:** *if I push with 1 Newton through 1 meter, I do work 1 Joule.*

- So lifting that kilogram took 10 Joules of work.

Only motion *in the direction of the force* counts …

- Carrying the weight straight across the room at constant height does no work on the weight.
- After all, it could have been just sliding across on ice—and the ice does no work!

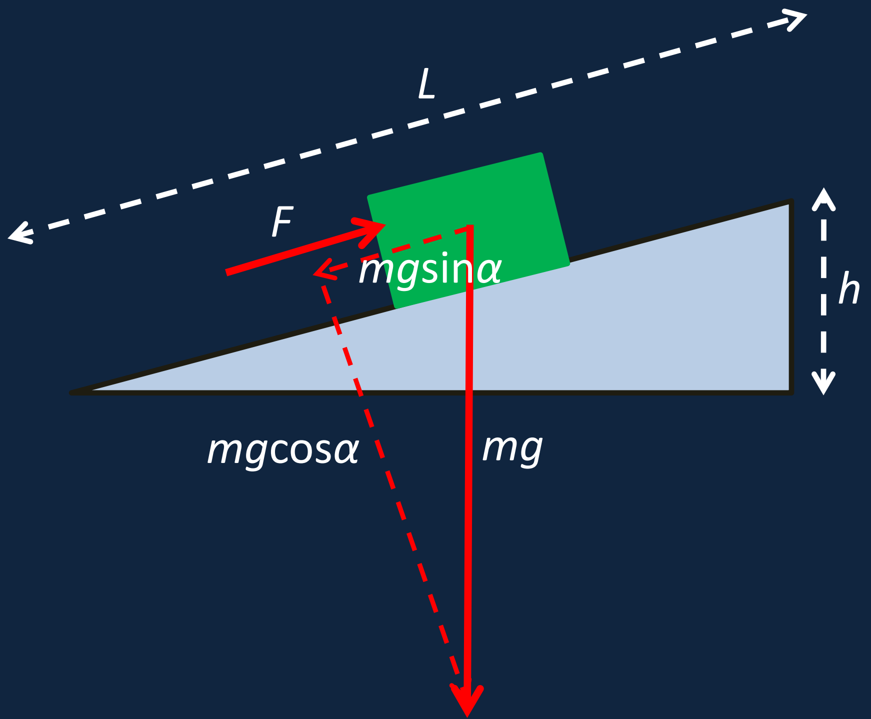
- What about pushing a box at constant velocity up a frictionless slope?

Pushing a box up a frictionless slope…

- Suppose we push a box of mass m at a steady speed a distance L up a frictionless slope of angle α.
- The work done is

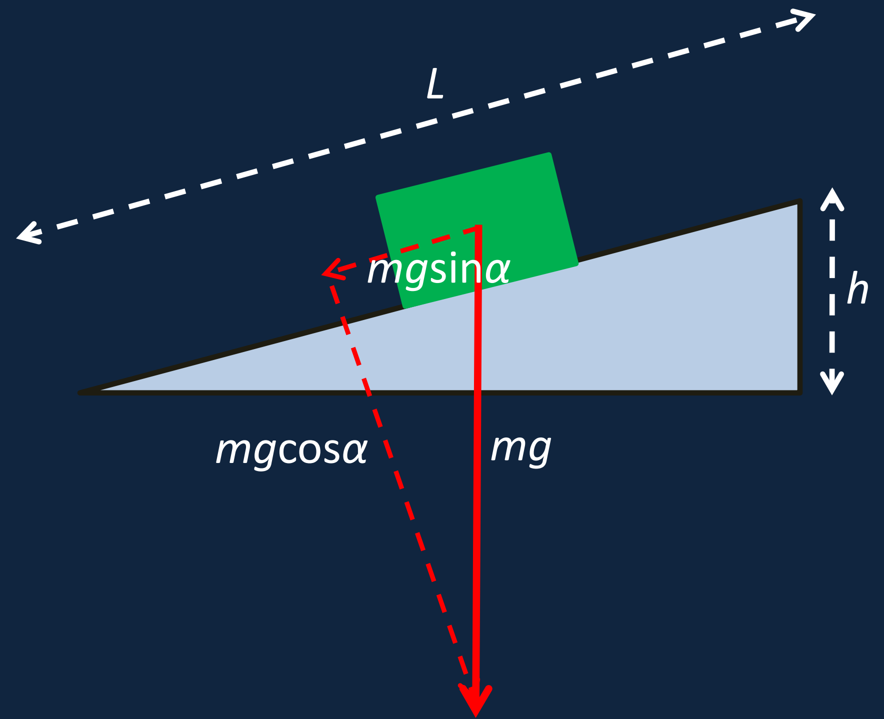
  $FL = mgL\sin\alpha = mgh$

  where h is the height gained.
- Meanwhile, gravity is doing *negative* work… its force is directed *opposite to the motion*.



…and letting it slide back down.

- Letting the box go at the top, the force of gravity along the slope, *mgL*sin*α,* will do exactly as much work on the box on the way down as we did pushing it up.
- Evidently, the work we did raising the box was *stored* by gravity.
- This "stored work" is called **potential energy** and is written $\boxed{U = mgh}$



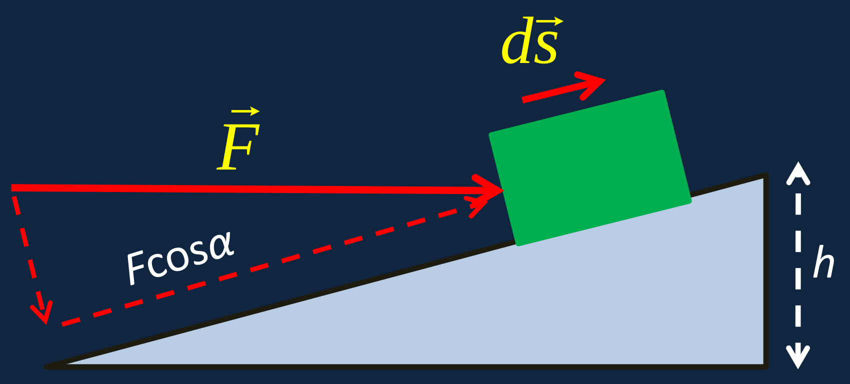
Energy is the Ability to Do Work

- We've established that pushing the box up a frictionless slope against gravity stores—in gravity—the ability to do work on the box on its way back down.
- This "stored work" is called potential energy.
- Notice it depends *not* on the slope, but only on the net height gained:

$$\boxed{U = mgh.}$$

What if you push the box *horizontally*?

- The box only moves up the slope, so only the component of force in that direction does any work.
- If the box moves a small distance *ds*, the work done
- $dW = (F\cos\alpha)ds$.
- This vector combination comes up a lot: we give it a special name… $dW = \vec{F} \cdot d\vec{s}$



The Vector Dot Product $\vec{A} \cdot \vec{B}$

- The dot product of two vectors is defined by:

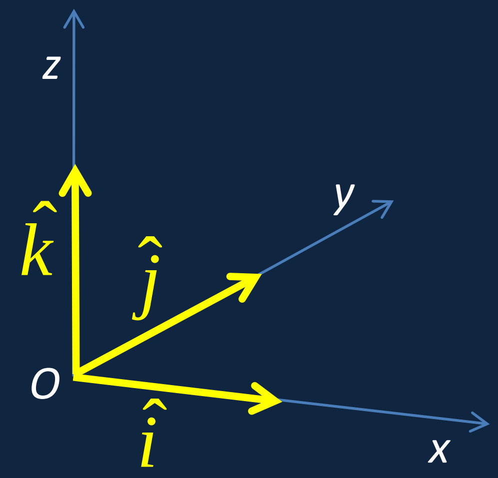
  $$\vec{A} \cdot \vec{B} = AB\cos\theta$$

  where *A*, *B* are the lengths of the vectors, and θ is the angle between them.

- Alternately: The dot product is the length of $\vec{A}$ multiplied by the length of the component of $\vec{B}$ in the direction of $\vec{A}$.
- From this $\vec{A} \cdot \left(\vec{B} + \vec{C}\right) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$.
- If the vectors are perpendicular, $\vec{A} \cdot \vec{B} = 0$.

Dot Product in Components

- Recall we introduced three orthogonal unit vectors $\hat{i}, \hat{j}, \hat{k}$ pointing in the directions of the *x*, *y* and *z* axes respectively.



- Note $\hat{i} \cdot \hat{i} = \hat{i}^2 = 1,\ \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0.$

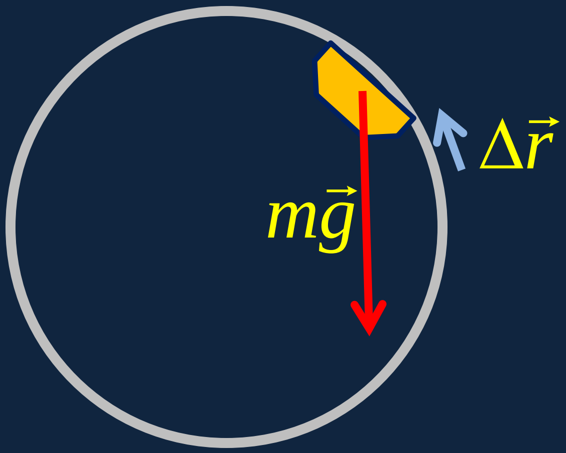
- Writing $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$ we find

$$\boxed{\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z.}$$

Positive and Negative Work

- As the loop the loop car climbs a small distance $\Delta\vec{r}$, the force of gravity $m\vec{g}$ does work $\vec{F}\cdot\vec{d} = m\vec{g}\cdot\Delta\vec{r}$. This is negative on the way up—the angle between the two vectors is more than 90°.
- Total work around part of the loop can be written

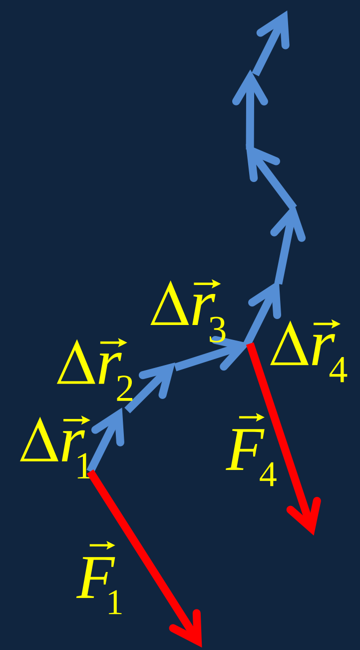
$$W = \sum \vec{F}\cdot\Delta\vec{r} = \int_{\vec{a}}^{\vec{b}} \vec{F}\cdot d\vec{r}$$



Work done by any Force along any Path

- The expression for work done along a path is general: just break the path into small pieces, add the work for each piece, then go to the limit of tinier pieces to give an integral:

$$W = \sum_i \vec{F}_i \cdot \Delta\vec{r}_i = \int_{\vec{a}}^{\vec{b}} \vec{F} \cdot d\vec{r}$$



In components:

$$W = \int_{a_x}^{b_x} F_x dx + \int_{a_y}^{b_y} F_y dy + \int_{a_z}^{b_z} F_z dz$$

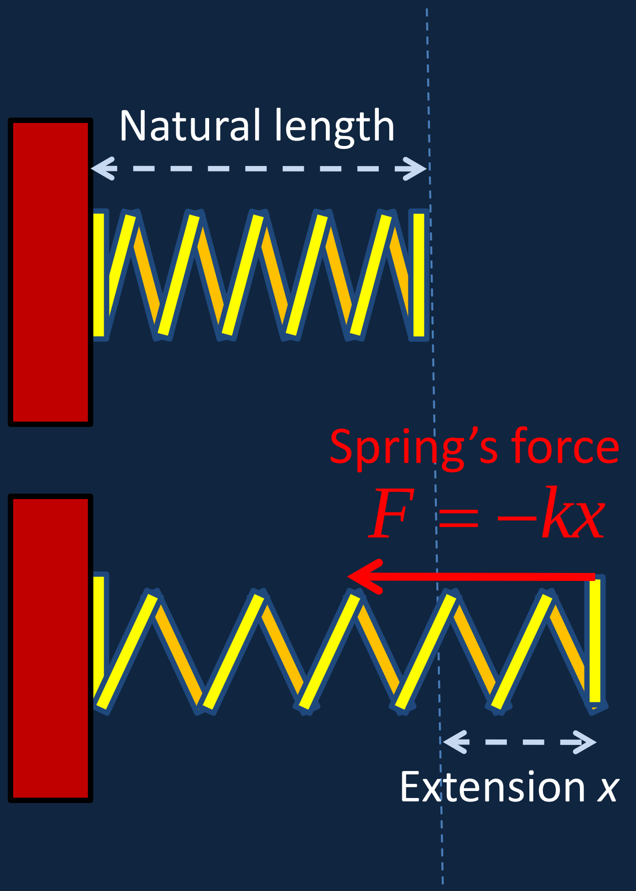
Force of a Stretched Spring

- If a spring is pulled to extend beyond its natural length by a distance *x*, it will pull back with a force

$$F = -kx$$

where *k* is called the "spring constant".

The same linear force is also generated when the spring is *compressed*.



Work done in *Stretching* a Spring

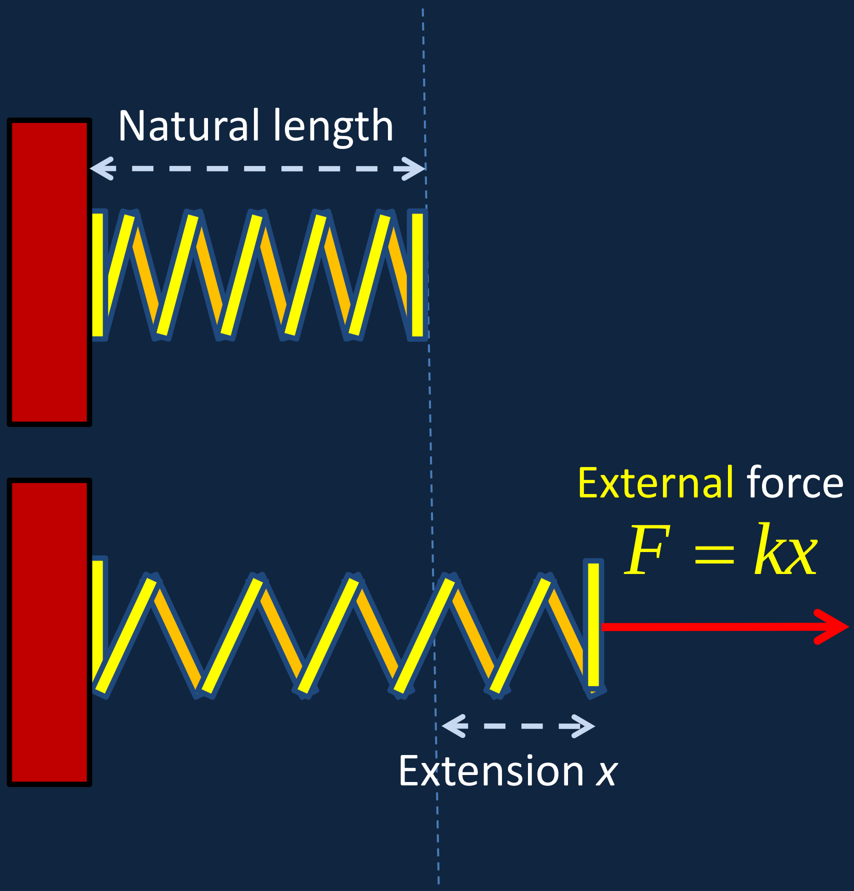
- The work from an external force needed to stretch the spring from

  x to $x + \Delta x$ is $kx\Delta x$,

  so the total work to stretch from the natural length to an extension x_0

$$W = \int_0^{x_0} kx\,dx = \tfrac{1}{2}kx_0^2.$$

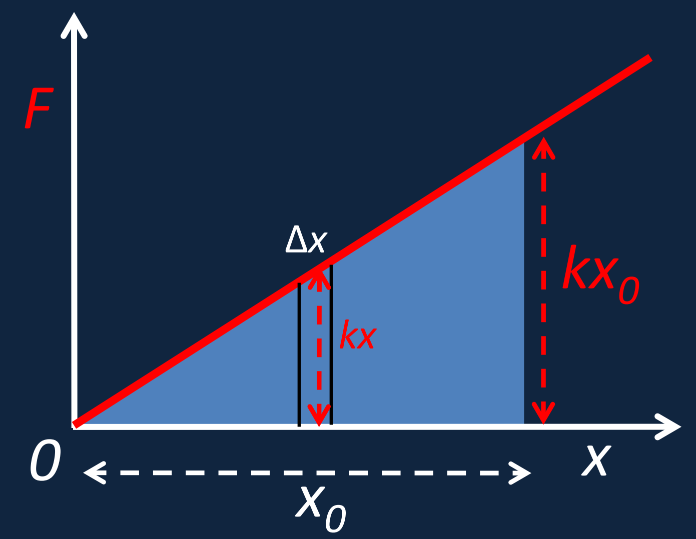
This work is stored by the spring as potential energy.



Total Work as *Area Under Curve*

- Plotting a graph of external force $F = kx$ as a function of *x*, the work to stretch the spring from x to $x + \Delta x$ is $kx\Delta x$, just the *incremental area under the curve*, so the total work is the total area

$$W = \int_0^{x_0} kx\,dx = \tfrac{1}{2}kx_0^2$$



Area under this "curve" =½ base x height= $\frac{1}{2}kx_0^2$
In fact, the total work done is the area under the force/distance curve for *any* curve: it's a sum of little areas $F\Delta x$ corresponding to work for Δx.